The Effects of Bank Size on Equilibrium Interest Rate Dispersion

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Introduction

- Why is there a dispersion of prices even for commodity goods in a given market?
- How do customers set their demand?
- Can non-homogenous prices become stable?

Models of Equilibrium Price Dispersion

Four different models of equilibrium price dispersion exist. They are:

1) Consumers have the same search costs and firms have same production costs

2) Consumers have different search costs, and firms have same production costs

3) Consumers have same search costs, and firms have different search costs

4)Consumers have different search costs and firms have different production costs

Market Visibility

- Market visibility is the chance a particular consumer, will visit a particular firm.
- In the different models of equilibrium price dispersion it is assumed that market visibility is constant across all firms.
- In this paper, the author uses a model with variable market visibility for firms, and uses it as a key factor in determining the equilibrium price dispersion.

Modeling Consumers

- Assume each consumer has their own search costs, but have the same inelastic demand and same willingness-to-accept.
- Also assume that all consumers knows the overall distribution of prices but not which banks offer which prices
- Consumers follow a sequential search rule. They will sample an interest rate from a bank and if the expected increase in interest rate is larger than their search cost, they will sample another bank. If not, the consumer will stop shopping and deposit money with the bank that quoted him the best price.
- Banks do not practice price discrimination per consumer.

Modeling Consumers

- Individual consumer's demand:

$$x_n = \begin{cases} \sum_{h=n+1}^{N} (r_h - r_n)\pi_h, & n < N \\ 0, & n = N \end{cases} \quad (1)$$

- Consumers with a per cost search of x will stop shopping when x<= xn

$$G(x) = \begin{cases} \frac{x}{sM}, & 0 \leq x \leq sM \\ 1, & x > sM \end{cases} \quad (2)$$

Consumer's Demand For Each Bank

- In order to derive the total demand for all banks, first must find the demand for bank1, which is the bank with the lowest interest rate

$$\begin{aligned} q_1 &= \int_{x_1}^{x^{max}} 1 \times \pi_1 g(x) M dx + \int_0^{x_1} 0 \times \pi_1 g(x) M dx \\ &= \pi_1 [G(x^{max}) - G(x_1)] M, \qquad (3) \\ \Longleftrightarrow \frac{q_1}{M} &= \pi_1 [G(x^{max}) - G(x_1)] \end{aligned}$$

- The demand for bank(n) is:

$$\frac{q_n}{M} = \begin{cases} \pi_n [G(x^{max}) - G(x_1)] + \pi_n \sum_{h=2}^{n} \frac{G(x_{h-1}) - G(x_h)}{1 - \sum_{k=1}^{n=1} \pi_k}, & n > 1 \\ \pi_n [G(x^{max}) - G(x_1)], & n = 1 \end{cases}$$

Consumer's Demand for Each Bank

- After substituting in the distribution of search costs and manipulating the n equations:

$$q_n = \pi_n[M - \frac{1}{s}(\bar{r} - r_n)], \text{ where } \bar{r} = \sum_{n=1}^{N} \pi_n r_n, n = 1, ..., N.$$

$$\Longleftrightarrow \frac{q_n}{M} = \pi_n[1 - \frac{1}{sM}(\bar{r} - r_n)]$$

- Taking the partials of the previous equation yields:

$$\frac{\partial q_n}{\partial r_d} = \frac{1}{sM} \begin{bmatrix} \pi_1(1-\pi_1) & -\pi_1\pi_2 & \dots & -\pi_1\pi_{N-1} & -\pi_1\pi_N \\ \pi_2\pi_1 & \pi_2(1-\pi_2) & \dots & -\pi_2\pi_{N-1} & -\pi_2\pi_N \\ \dots & \dots & \dots & \dots & \dots \\ \pi_{N-1}\pi_1 & -\pi_{N-1}\pi_2 & \dots & \pi_{N-1}(1-\pi_{N-1}) & -\pi_{N-1}\pi_N \\ \pi_N\pi_1 & -\pi_N\pi_2 & \dots & -\pi_N\pi_{N-1} & \pi_N(1-\pi_N) \end{bmatrix}$$

Bank Profit Maximization

- Assumptions made by the model:
  - Banks have separate, mutually exclusive markets for their deposit and loan accounts.
  - Banks take into consideration their costs of production, the opportunity costs, and the pricing behavior of other banks, when pricing their accounts.
  - National rates for both deposit and loan accounts factor in the local market.
  - Banks have correct information about how other banks will respond to given changes in the market.

Bank Profit Maximization

- Each bank maximizes profits by:
  - Maximizing return on loans and funds invested
  - Minimizing the cost of deposits and other liabilities
  - Taking consumer search behavior, expected demand, response from other banks as givens.

Bank Profit Maximization

- The variables used in the derivation of bank profit maximization are:

c_{nd} as the non-interest variable cost of processing account d,

c_{ns} as the non-interest variable cost of processing securities,

S_n the balance of securities,

C_n as fixed production costs,

K_n as total bank capital,

$\hat{\mathbf{r}}_\mathbf{d} = [r_{1d}\ r_{2d}\ \dots\ r_{Nd}]'$ as the vector of market prices for the dth product,

$\hat{\mathbf{r}}_\mathbf{n} = [r_{n1}\ r_{n2}\ \dots\ r_{n(D+L)}]'$ as the vector of Bank n's prices, and

$q_{nd}(\hat{\mathbf{r}}_\mathbf{d})$ as Bank n's expected demand, as a function of the vector of market rates for account d.

Maximization of Single Bank’s Profits

- Each bank wants to maximize it’s own vector of interest rates, R, that maximize profits.
- An individual bank’s profit maximization is:

$$\text{Max } \Pi^i = \sum_{l=L+1}^{D+L} (r_{nl} - c_{nl}) q_{nl}(\hat{\mathbf{r}}_{\mathbf{l}}) + (r_s - c_{ns}) S_n$$
$$- \sum_{d=1}^{(} D)(r_{nd} + c_{nd}) q_{nd}(\hat{\mathbf{r}}_{\mathbf{j}}) - C_n,$$

- This is bounded by the fact that assets equals liabilities:

$$\sum_{l=L+1}^{D+L} q_{nl}(\hat{\mathbf{r}}_{\mathbf{j}}) + S_n = (1 - \rho) \sum_{d=1}^{D+L} q_{nd}(r_{nd}) + K_n$$

Maximization of All Bank's Profits

- Solving for the Bank N's maximization:

$$\begin{aligned}
\text{Max } \Pi^n &= \sum_{l=D+1}^{D+L} (r_{nl} - c_{nl}) q_{nl}(\hat{\mathbf{r}}_{\mathbf{l}}) \\
&+ (r_s - c_{ns}) \left((1-\rho) \sum_{d=1}^{D} q_{nd}(r_{nd}) K_n - \sum_{d=1}^{D+L} q_{nd}(\hat{\mathbf{r}}_{\mathbf{d}}) \right) \\
&- \sum_{d=1}^{D+L} (r_{nd} + c_{nd}) q_{nd}(\hat{\mathbf{r}}_{\mathbf{d}}) - C_n,
\end{aligned}$$

Equilibrium Interest Rate Dispersion

- Assumptions of model:
  - Given the market demand, bank profit maximization, and market supply for a given account, the equilibrium price and quantity can be found.
  - The equilibrium price will be a distribution of interest rates that are closely linked to the distribution of market presence.

Equilibrium Interest Rate Dispersion

- In order to solve for the equilibrium prices and quantities the model:
  - Make two definitions
  - Prove a proposition using three claims:
    - Claim 1 shows that the lowest offered rate in the market is r min, otherwise banks could all drop prices simultaneously.
    - Claim 2 shows that there cannot be a single rate equilibrium.
    - Claim 3 is that a equilibrium distribution of prices exists

Equilibrium Interest Rate Dispersion

- Definitions used to prove an equilibrium price distribution exists:

Definition 1: Given a set of account d market output quantities $< q_{nd} >_{n=1}^{N}$ and the reservation price $r_{d\,min}$, a *bank equilibrium* is the set of both price distribution $F(\cdot)$ and bank profits $< \Pi^n >_{n=1}^{N}$ such that $\Pi^n \geq \Pi^n(r_{nd})$ for all r_{nd} in the support of $F(\cdot), n = 1, ..., N$.[8]

Definition 2: The triple $(F(\cdot), < \Pi^n >_{n=1}^{N}, < q_{nd} >_{n=1}^{N})$ is a *market equilibrium* if and only if for some fixed $r_{d\,max}$ and distribution of search costs $G(\cdot)$, (a) $(F(\cdot), < \Pi^n >_{n=1}^{N})$ is a bank equilibrium and (b) expected demand $< q_{nd} >_{n=1}^{N}$ is generated from the cost-minimizing strategies of consumers facing $F(\cdot)$.

Equilibrium Interest Rate Dispersion

- The proposition, aka inequality to must be solved to yield an equilibrium interest rate distribution:

$$\sum_{n=2}^{N} \pi_n (r_{nd} - r_{2d}) < sM < (1 - \pi_n) \left(\frac{(r_s - c_{ns}) - (r_{nd} + c_{nd})}{1 - \frac{1}{sM} (\bar{r}_d - r_{nd})} \right)$$

- In order to prove that if this inequality holds, there will be an equilibrium interest rate dispersion, the following three claims are made.

Equilibrium Interest Rate Dispersion

- Claim 1: If there exists a single-rate market equilibrium, then r1=rdMin.
- Claim 2: There can be no single price market equilibrium
- Claim 3: Given a maximum cost of search sM that satisfies

$$\sum_{n=2}^{N} \pi_n (r_{nd} - r_{2d}) < sM < (1 - \pi_n) \left(\frac{(r_s - c_{ns}) - (r_{nd} + c_{nd})}{1 - \frac{1}{sM}(\bar{r}_d - r_{nd})} \right)$$

no bank will deviate its price from the non-degenerate distribution of interest rates

Discussion

- Market presence has at least three interesting and independent effects on the equilibrium distribution of prices offered in the market.
  - Banks with relatively poor rates but good presence can capture the same market share of accounts as banks with the opposite set of conditions
  - Banks with large market presence benefit most both from offering better rates and from the rate worsening by their rivals.
  - Market presence and search costs determine the existence and uniqueness of the equilibrium.

Conclusions

- The amount of market presence greatly influences market share
- Banks should try to balance having high costs of obtaining a larger market presence, given the fact the higher market presence will lead to higher market share.