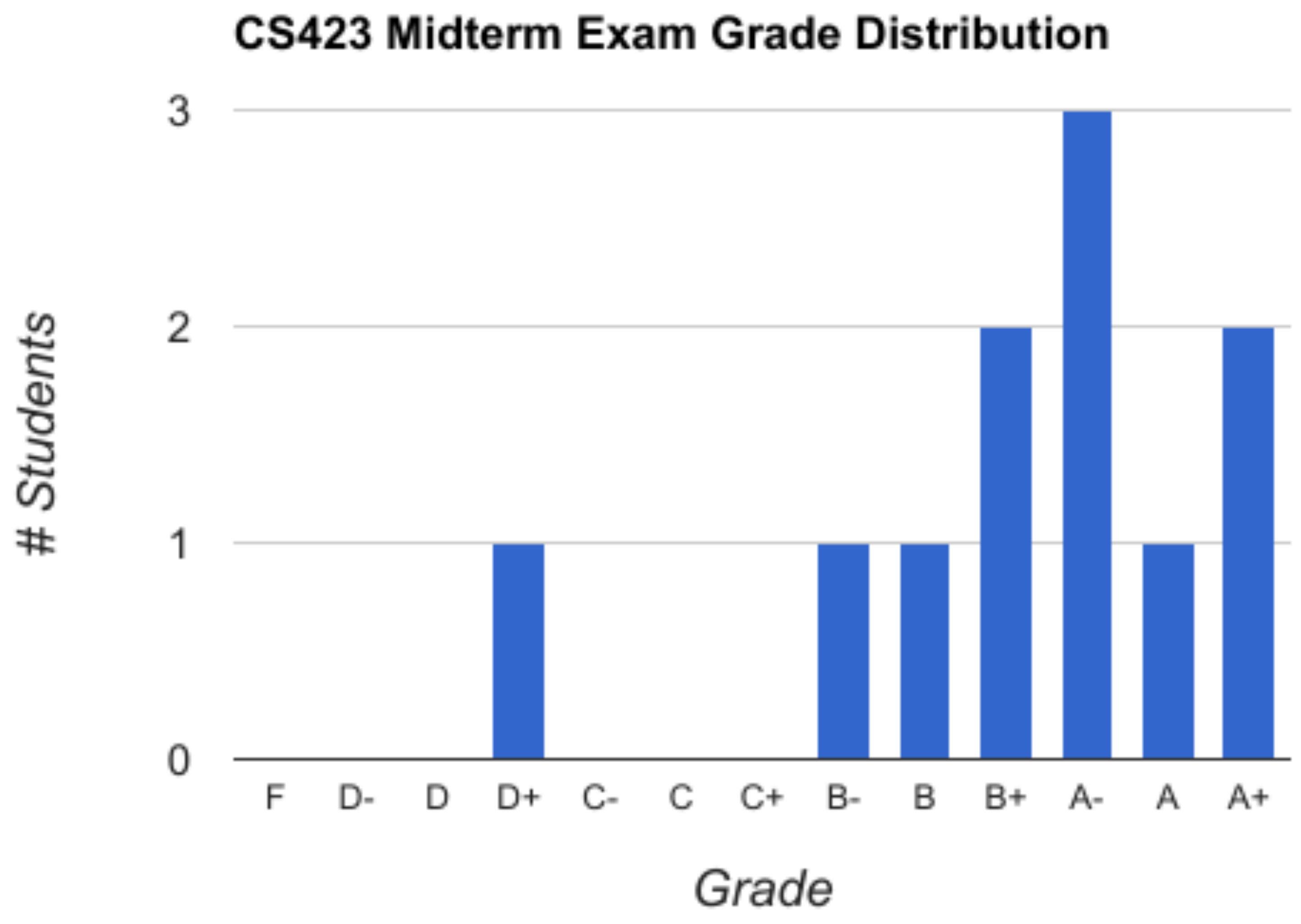
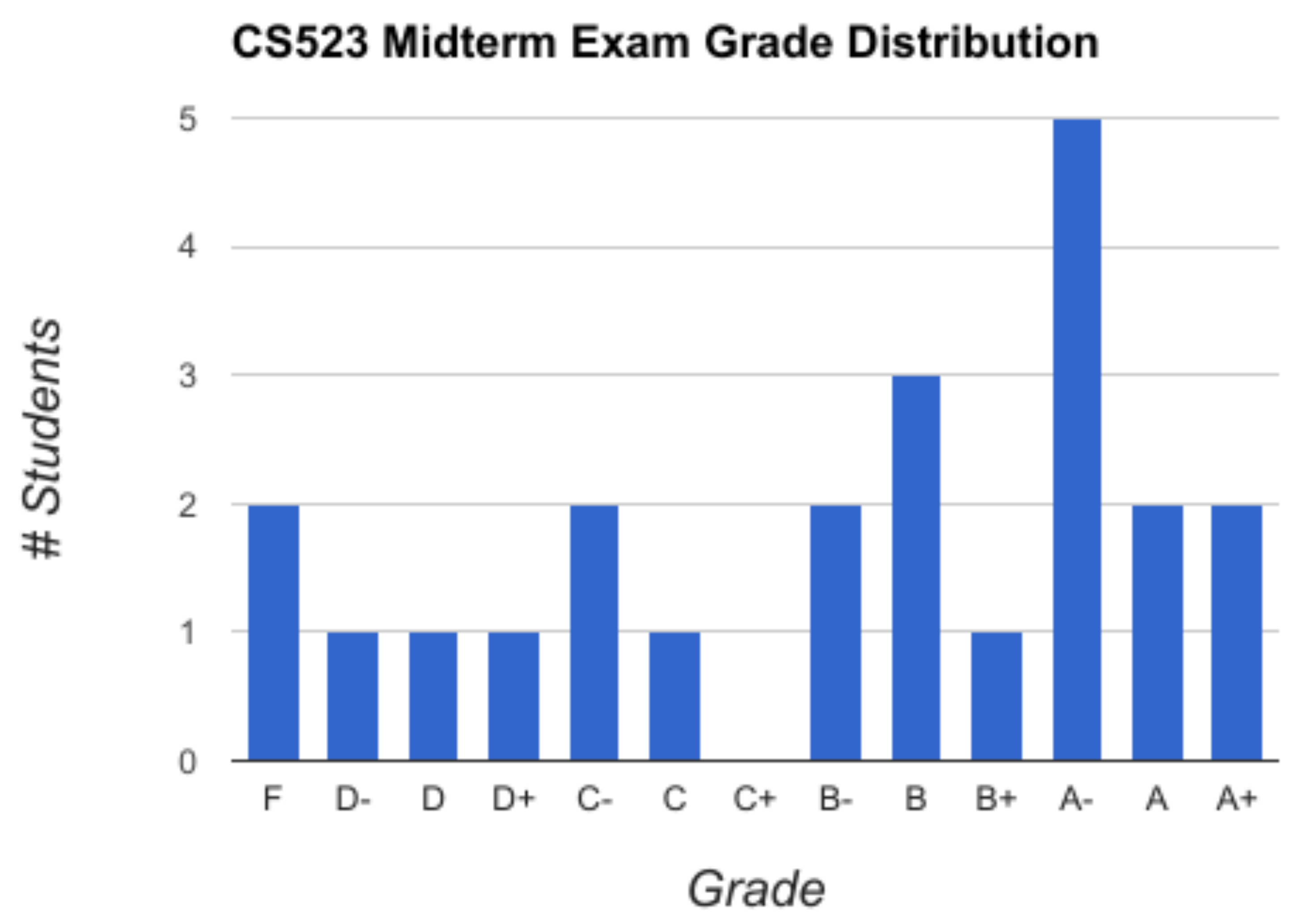
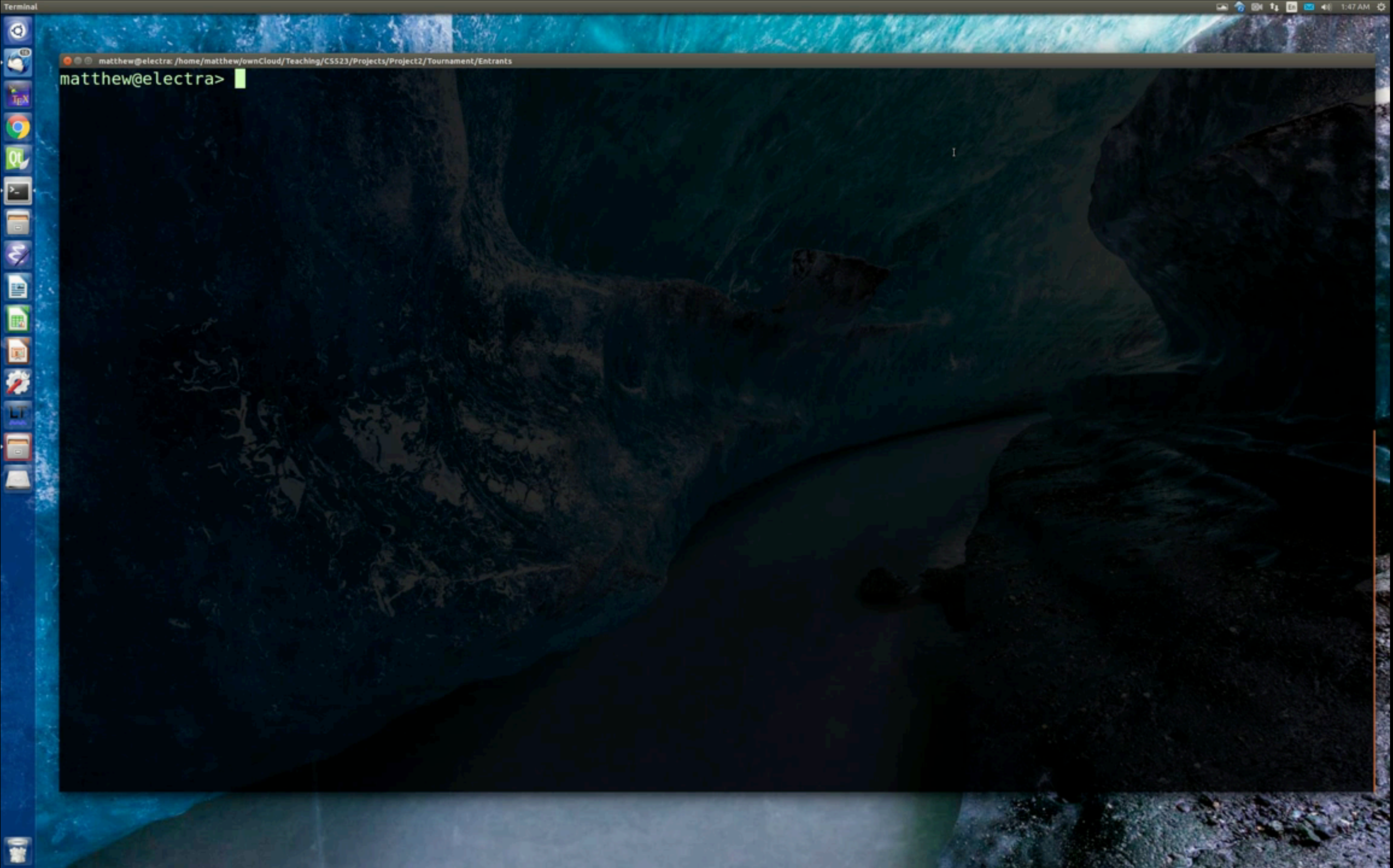
Logistics

- Grades for the midterm and the course average have been posted on learn.unm.edu.
- Reviews are due today at 6:00pm
- Project 3 will be posted this afternoon.
- Self organise groups of 2 for Project 3.
- Project 4 will likely be individual.
- Transitioning to Modelling and Game Theory this week.


CS423 Midterm Exam Grade Distribution
Students
3
2
1
0
F
D-
D
D+
C-
C
C+
B-
B
B+
A-
A
A+
Grade


CS523 Midterm Exam Grade Distribution
Students
Grade
0
1
2
3
4
5
F
D-
D
D+
C-
C
C+
B-
B
B+
A-
A
A+

Core Wars Class Competition


Terminal
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matthew@electra>

Terminal

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matthew@electra>
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Terminal
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matthew@electra>
2:19 AM

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matthew@electra>
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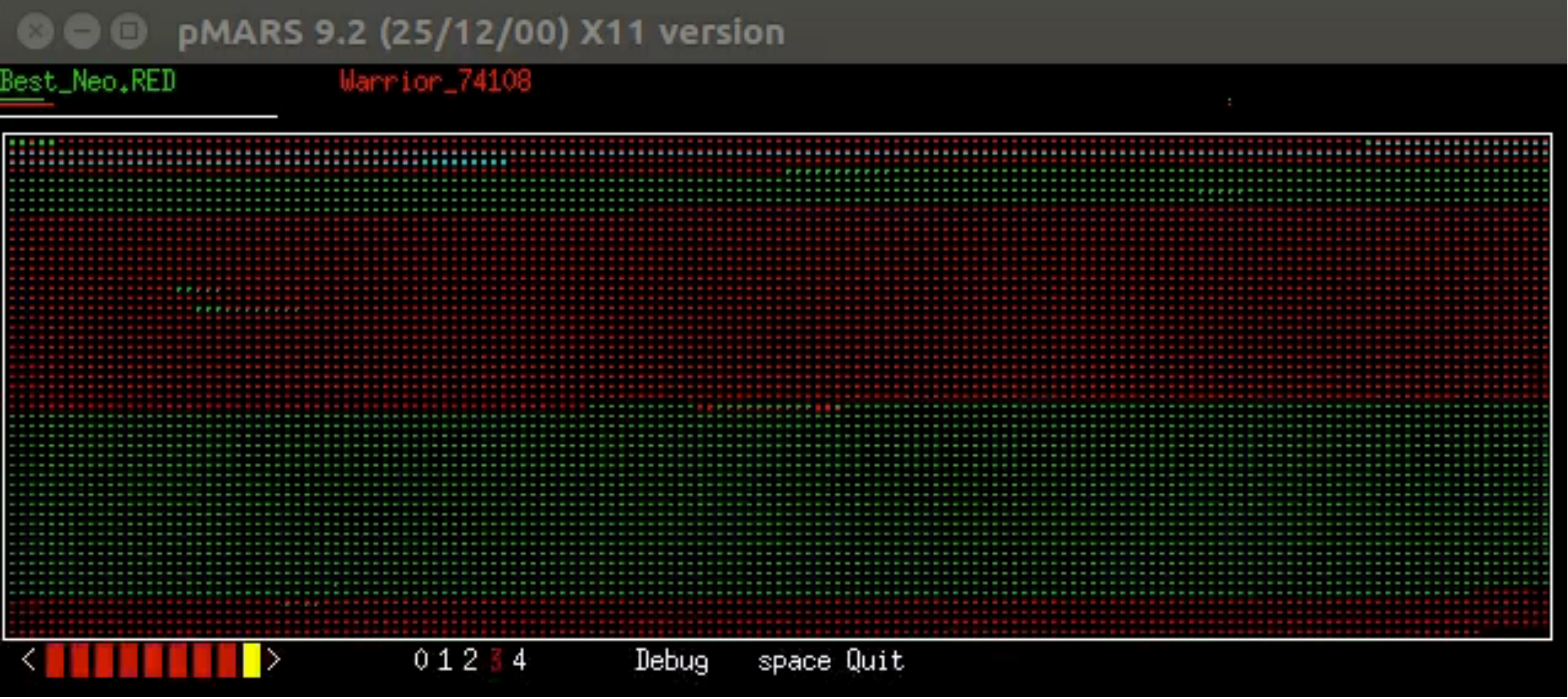
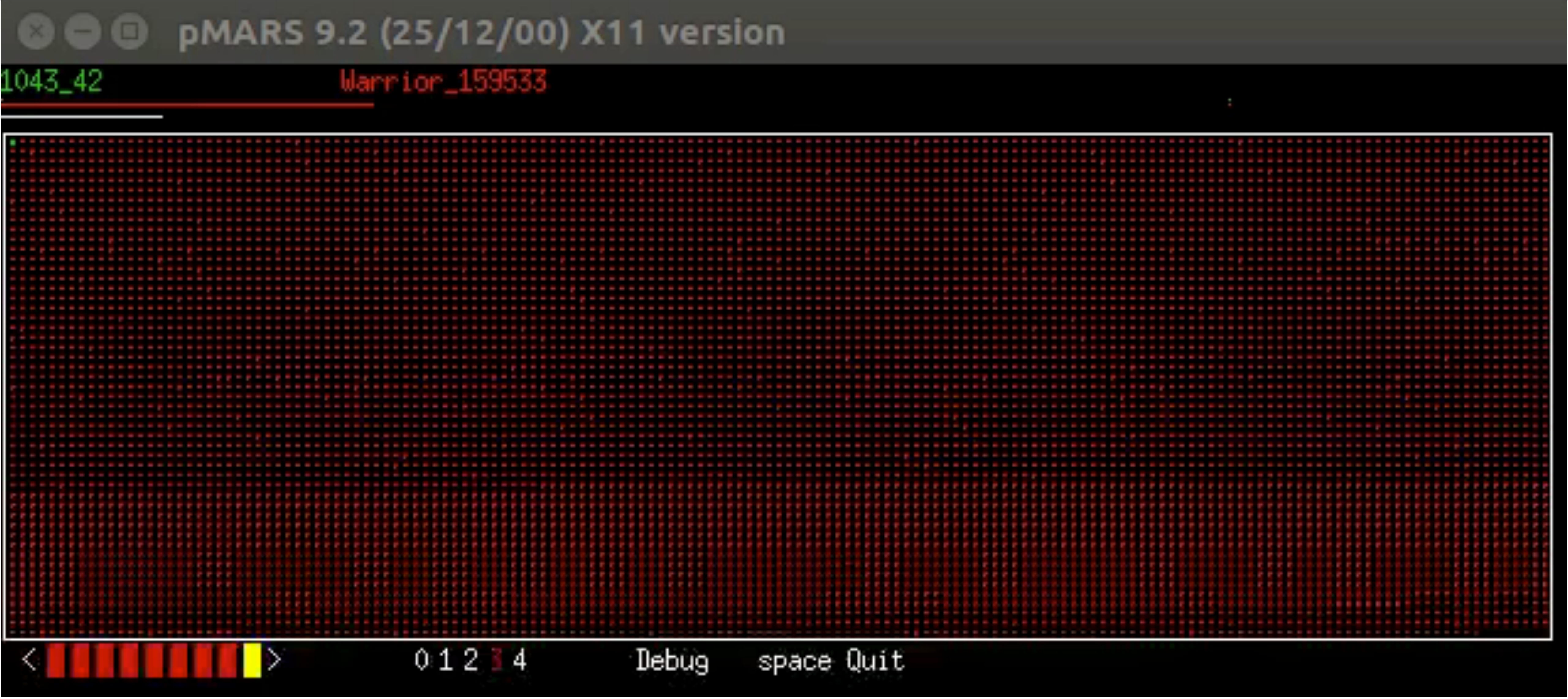


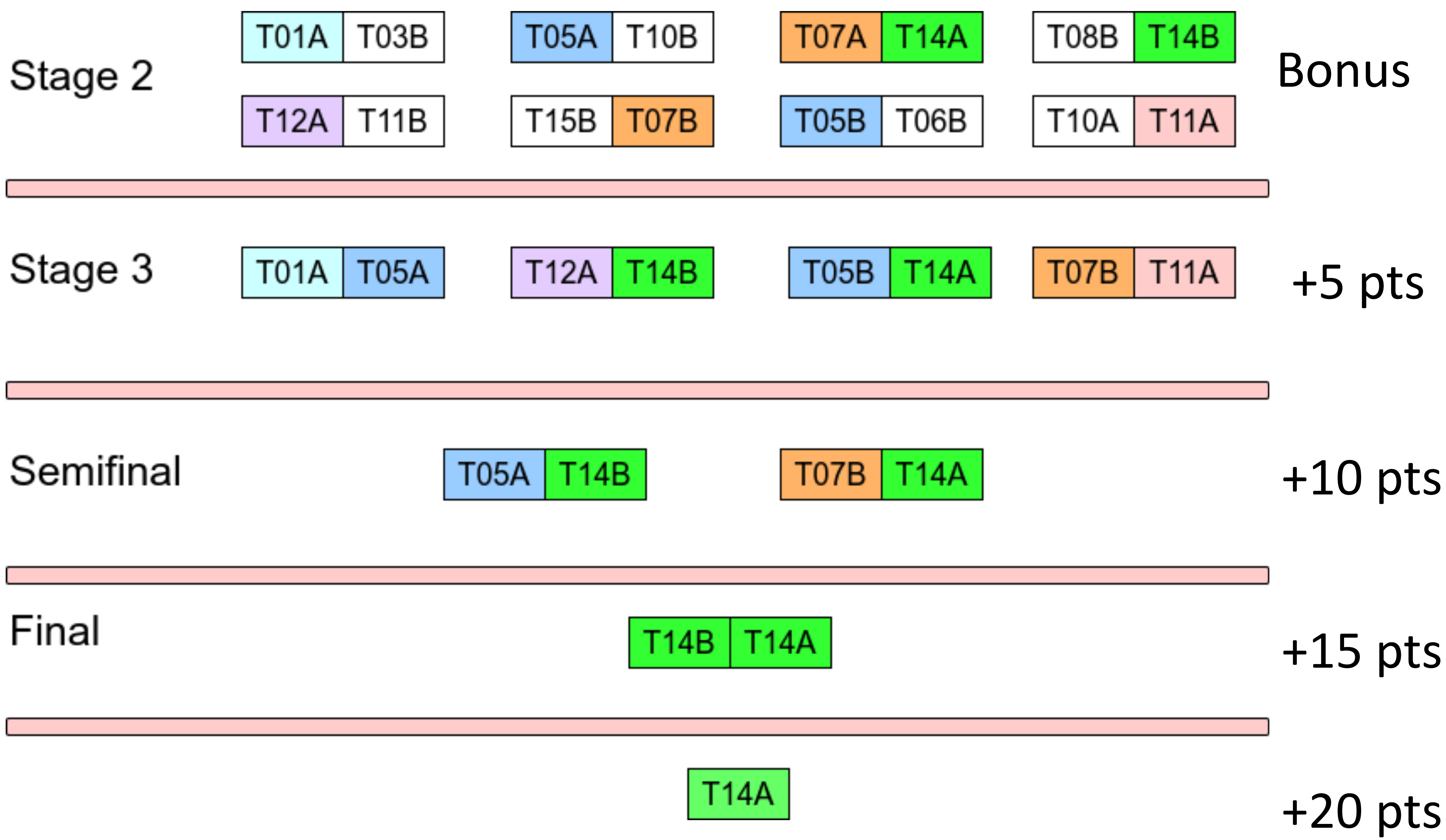
Semifinal: Team 7 vs Team 14 A

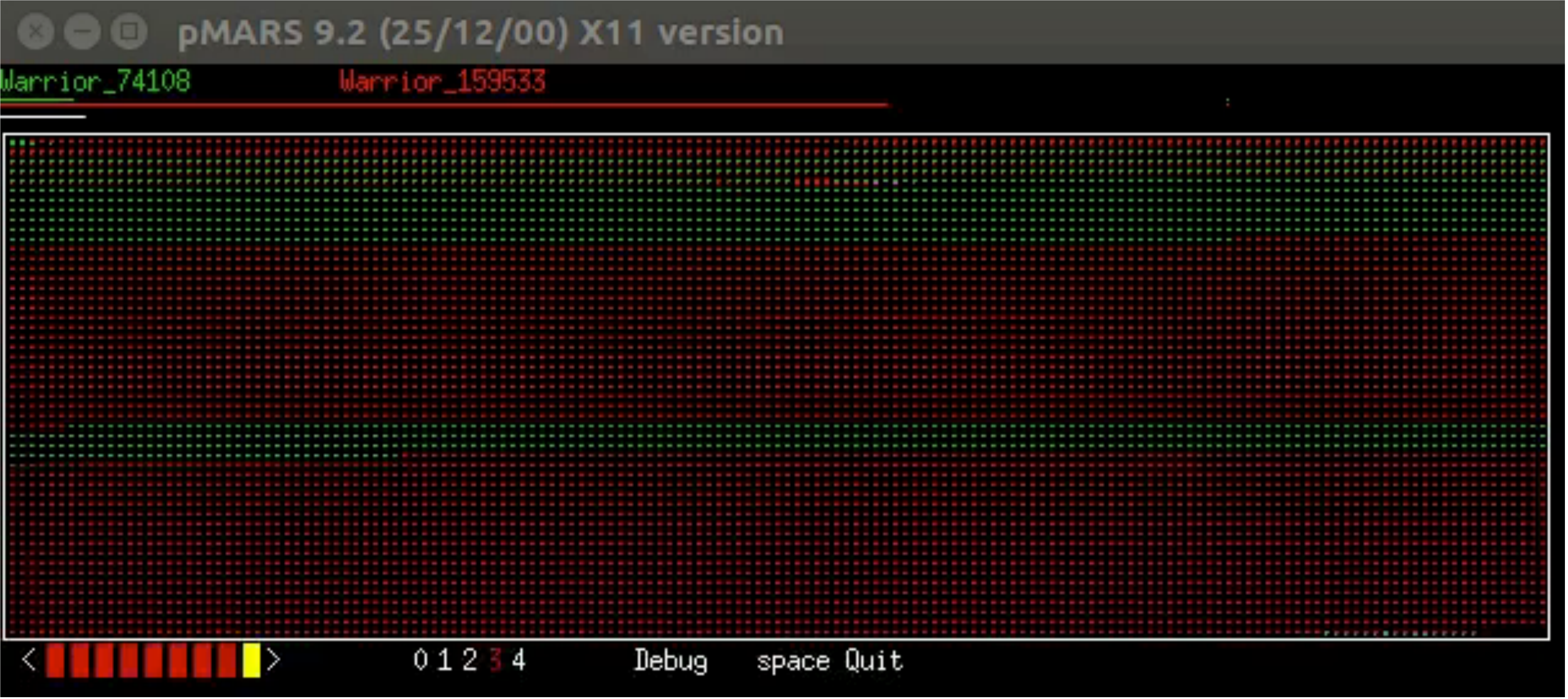


Semifinal: Team 5 vs Team 14 B

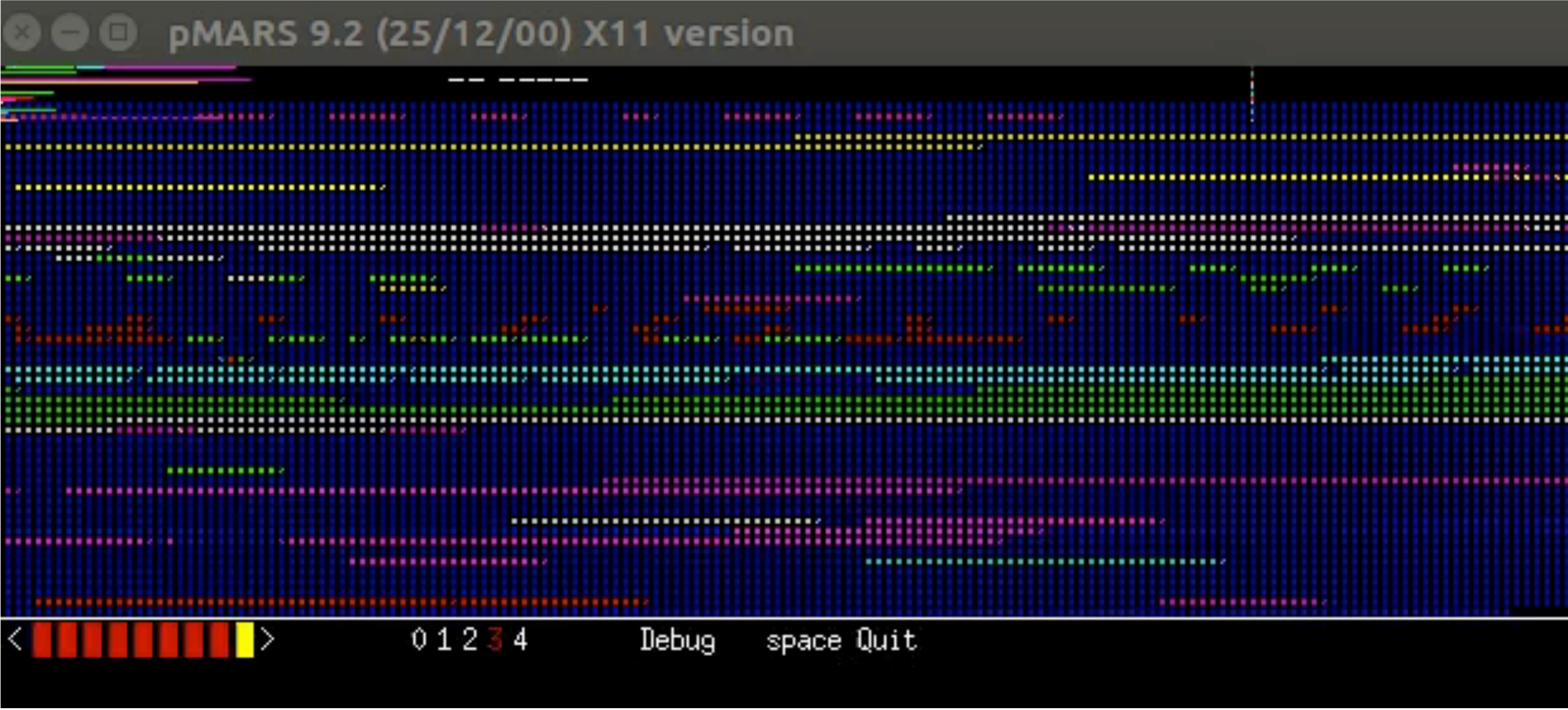
Warriors 14 A and B move to the final!

Serena (16) and Venus Williams (17), Australian Open 1st and 2nd place.


Stage 2
T01A T03B
T05A T10B
T07A T14A
T08B T14B
T12A T11B
T15B T07B
T05B T06B
T10A T11A
Bonus
Stage 3
T01A T05A
T12A T14B
T05B T14A
T07B T11A
+5 pts
Semifinal
T05A T14B
T07B T14A
+10 pts
Final
T14B T14A
+15 pts
T14A
+20 pts



Final: Warrior 14A vs 14B (+35 bonus points)



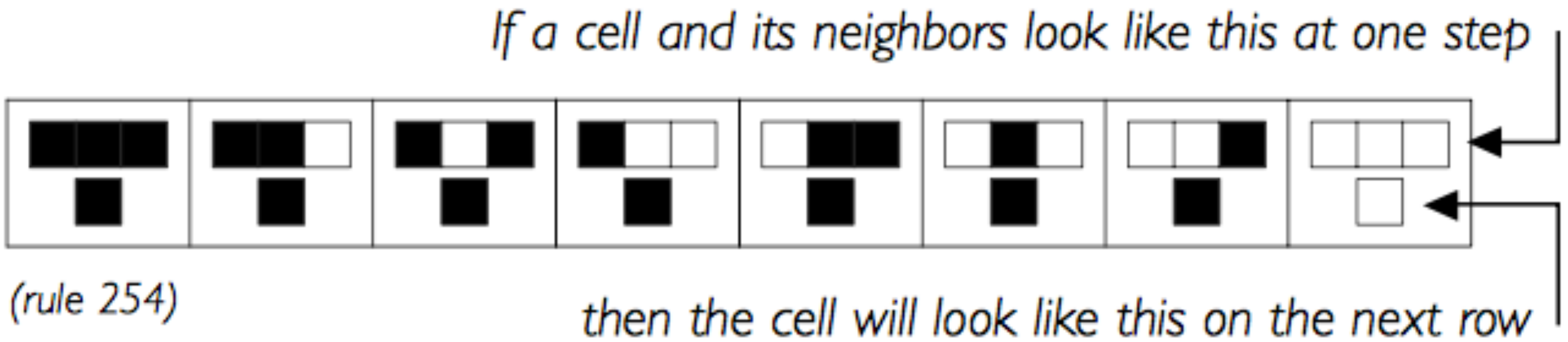
Bonus Round: Free for All

matthew@electra: /home/matthew/ownCloud/Teaching/CS523/Projects/Project2/Tournament/Entrants

+10 pts

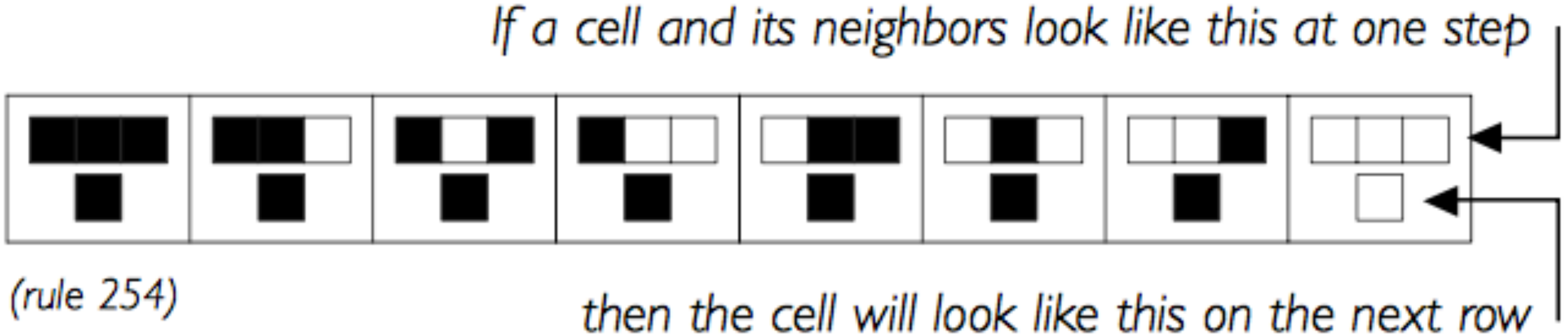
```
      MOV.I   *      8, *      5
      SUB.AB #      8, *      7

b21911d9-4ae3-396a-9d54-a2c9b1370424 by T17 scores 49624
 Results: 0 0 0 0 0 0 0 0 1 2 3 2 6 9 5 21 24 28 38 51 74 105 129 105 112 76 97 55 24 6 0 0 0 0 28
: T10-blu-solly by : G10 scores 49287
 Results: 0 0 0 0 1 0 0 1 0 2 3 3 4 7 2 20 25 26 36 52 73 104 131 106 113 76 97 55 24 6 0 0 0 0 34
: T10-red-solly by : G10 scores 49267
 Results: 0 0 0 0 1 0 0 1 0 1 4 3 3 7 3 19 26 28 38 55 72 100 128 106 113 76 97 55 24 6 0 0 0 0 35
DD_203.red by Damion Terrell and Dominic Paul Delvecchio scores 49056
 Results: 0 0 0 0 1 0 0 0 0 1 2 3 5 7 5 19 27 28 39 50 71 102 130 105 112 76 95 55 24 6 0 0 0 0 38
c251438d-6d43-36fd-8ff2-0fb8bd5c9f4e by T17 scores 48970
 Results: 0 0 0 0 0 0 0 1 0 0 3 2 6 7 4 19 24 29 37 53 71 104 128 106 113 76 97 55 24 6 0 0 0 0 36
Unknown by Anonymous scores 48669
 Results: 0 0 0 0 0 0 0 1 0 2 4 3 3 9 4 20 23 25 37 53 72 102 126 103 113 74 97 55 24 6 0 0 0 0 45
DD_204.red by Damion Terrell and Dominic Paul Delvecchio scores 47865
 Results: 0 0 0 0 1 0 0 1 0 0 4 1 5 6 4 16 20 25 37 52 71 102 128 104 111 75 96 55 24 6 0 0 0 0 57
Warrior_159533 by T14 scores 47682
 Results: 0 0 0 0 0 0 0 1 1 2 0 2 4 7 3 18 22 28 36 50 72 98 128 105 109 74 97 55 24 6 0 0 0 0 59
Warrior_74108 by T14 scores 47518
 Results: 0 0 0 0 0 0 0 0 0 2 1 3 3 8 4 19 23 24 37 50 69 100 128 103 111 74 96 55 24 6 0 0 0 0 61
Unknown by Anonymous scores 47495
 Results: 0 0 0 0 0 0 0 0 0 2 2 2 4 7 4 19 24 22 35 50 70 99 129 105 109 75 97 55 24 6 0 0 0 0 61
:  CURRENT_WARRIOR_NAME by :  GROUP_T9 scores 47045
 Results: 0 0 0 0 0 0 0 0 0 1 2 2 5 7 1 16 19 30 35 50 69 100 126 104 111 74 97 55 24 6 0 0 0 0 67
:  CURRENT_WARRIOR_NAME by :  GROUP_T9 scores 46597
 Results: 0 0 0 0 0 0 0 1 0 1 1 0 3 5 4 15 22 28 31 53 67 98 126 106 110 76 96 55 24 6 0 0 0 0 73
~999[706] by T1 scores 46263
 Results: 0 0 0 0 0 0 0 0 1 0 3 1 2 6 2 18 18 27 37 50 69 98 124 98 109 75 97 55 24 6 0 0 0 0 81
~999[424] by T1 scores 46243
 Results: 0 0 0 0 0 0 0 0 1 0 2 2 3 6 3 12 21 26 35 51 68 95 126 104 110 75 97 54 24 6 0 0 0 0 80
Bester_Neo.RED by Butterfly Wings scores 45286
 Results: 0 0 0 0 0 0 0 0 0 1 2 2 3 4 4 15 22 23 32 43 68 101 117 103 112 71 96 54 24 6 0 0 0 0 98
Best_Neo.RED by Butterfly Wings scores 44702
 Results: 0 0 0 0 0 0 0 0 0 1 1 2 3 4 4 13 20 22 35 43 64 94 126 101 109 75 94 53 24 6 0 0 0 0 107
: T3-1st-place-warrior.RED by : T3 scores 43857
 Results: 0 0 0 0 1 0 0 0 0 0 1 0 1 3 2 17 16 20 24 45 67 98 120 104 109 74 95 55 24 6 0 0 0 0 119
```

Cellular Automata

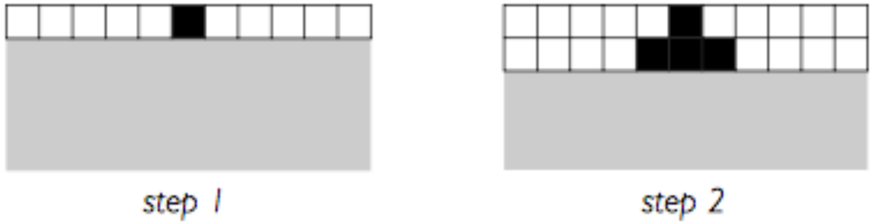


Wolfram, S., A New Kind of Science

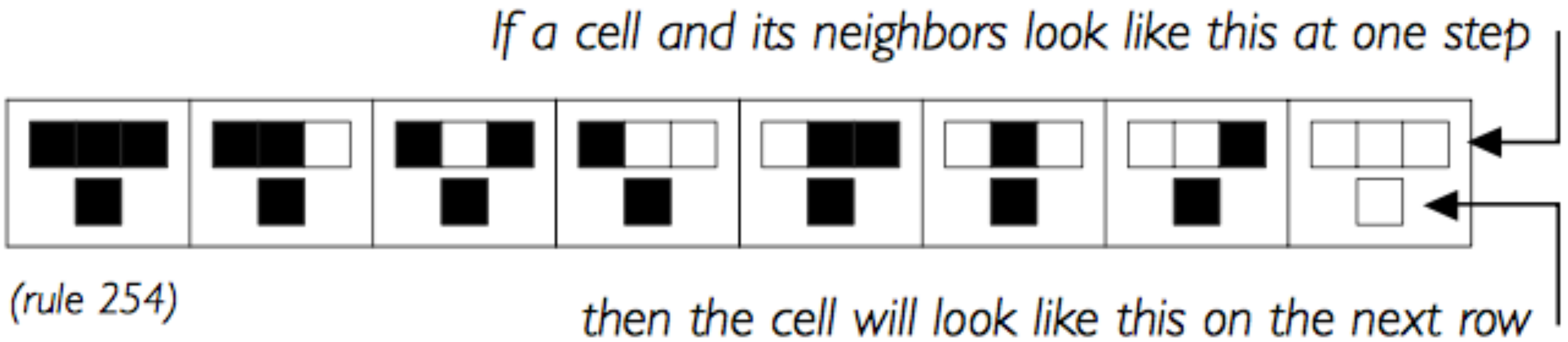
Cellular Automata



With the rule above, a simple pattern is produced.



Cellular Automata



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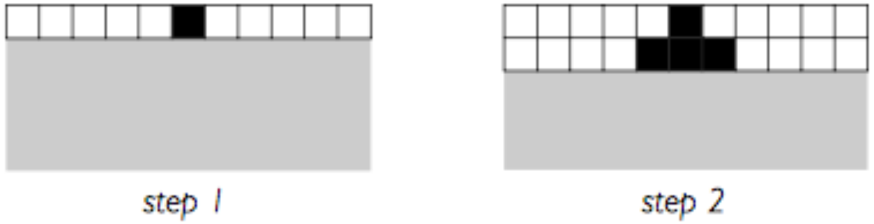
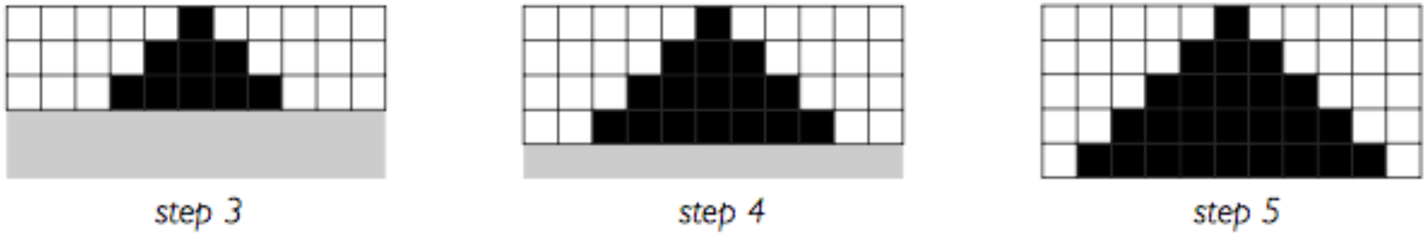




Cellular Automata

Wolfram's Rule Numbering System.

We keep the order of input combinations fixed.

Our numbering is just the decimal for the output bits:

111
110
101
100
011
010
001
000

Cellular Automata

Wolfram's Rule Numbering System.

We keep the order of input combinations fixed.

Our numbering is just the decimal for the output bits:

$$\begin{matrix} 111 \\ 110 \\ 101 \\ 100 \\ 011 \\ 010 \\ 001 \\ 000 \end{matrix} \rightarrow \begin{matrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{matrix}$$

Cellular Automata

Wolfram's Rule Numbering System.

We keep the order of input combinations fixed.

Our numbering is just the decimal for the output bits:

$$\begin{matrix} 111 \\ 110 \\ 101 \\ 100 \\ 011 \\ 010 \\ 001 \\ 000 \end{matrix} \rightarrow \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{matrix} = 11111110 = 254$$

Cellular Automata

Wolfram's Rule Numbering System.

We keep the order of input combinations fixed.

Our numbering is just the decimal for the output bits:

$$\begin{matrix} 111 \\ 110 \\ 101 \\ 100 \\ 011 \\ 010 \\ 001 \\ 000 \end{matrix} \rightarrow \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{matrix} = 00011110 = 30$$

Cellular Automata

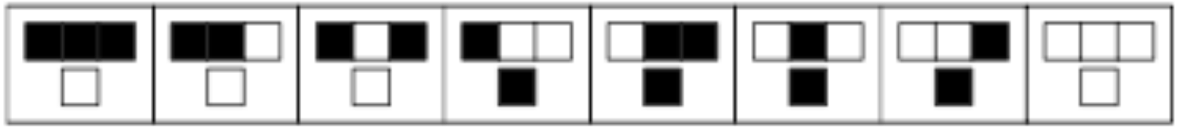
Wolfram's Rule Numbering System.

We keep the order of input combinations fixed.

Our numbering is just the decimal for the output bits:

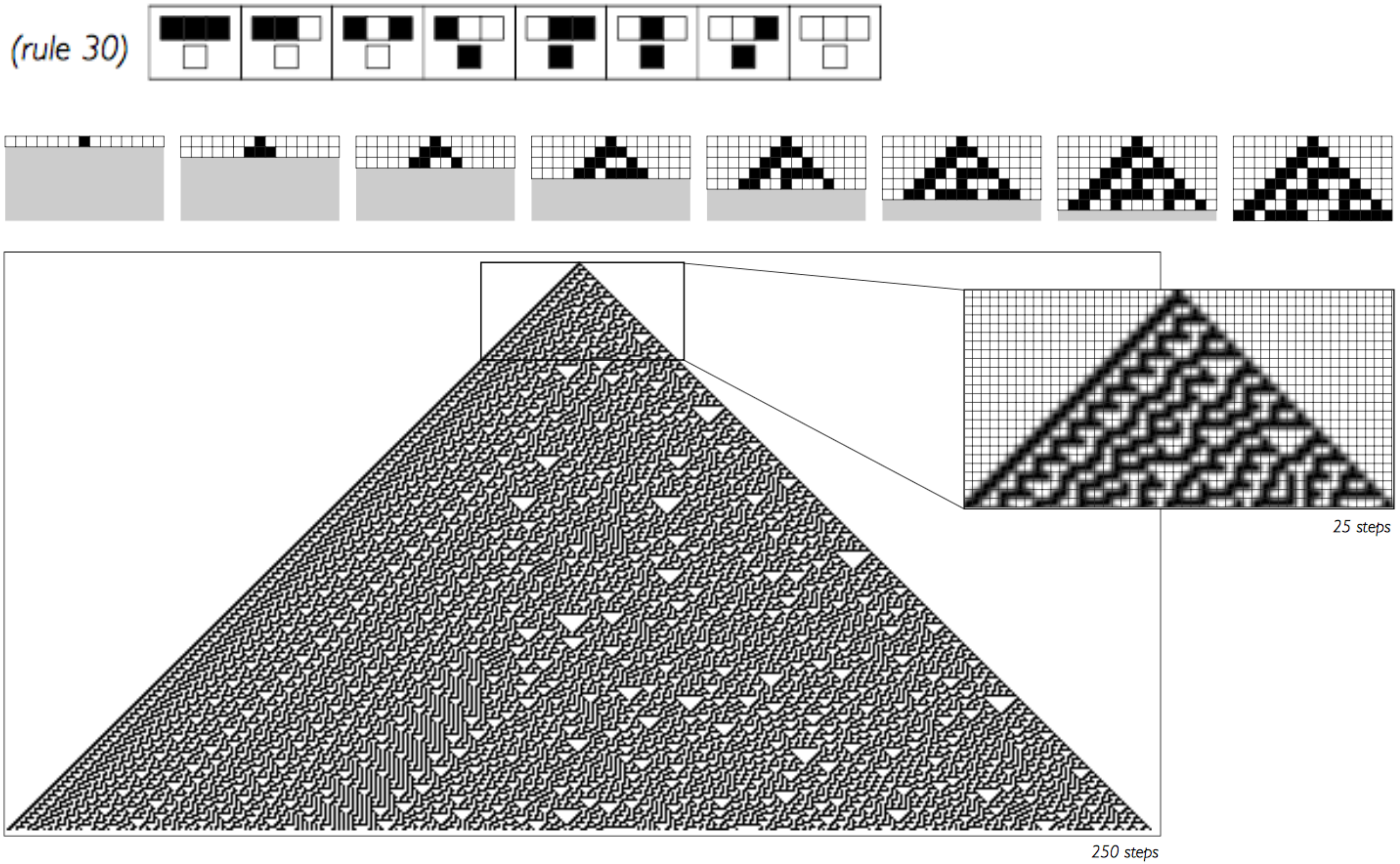
$$\begin{array}{l} 111 \\ 110 \\ 101 \\ 100 \\ 011 \\ 010 \\ 001 \\ 000 \end{array} \rightarrow \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{array} = 00011110 = 30$$

(rule 30)



Number of Elementary CAs

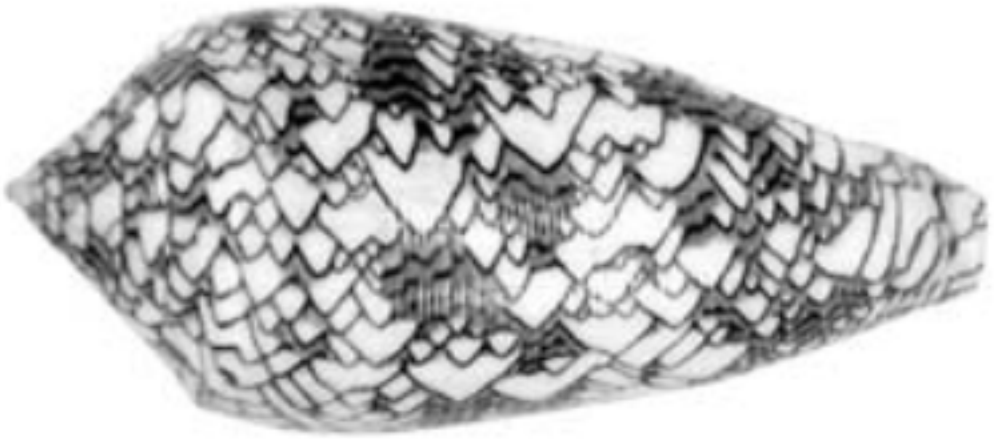
- We have seen that there are 256 unique rules.
- We can reduce this number to 128 because of the 1/0 symmetry.
- We can further reduce this number to 64 through left/right symmetry.


(rule 30)
25 steps
250 steps

Cellular Automata

Seashell pattern

Cellular Automata



Seashell pattern

We are back to our recurring theme in this class. Simple rules can produce very complex behaviour.
The complexity of the world (computational and physical) can be modelled with simple rules, and that
This may be the **only** way to study many systems.

Universal Computation

- One of the most powerful systems we know of is the Universal Turing Machine (The subject of most of CS500).

- Each Turing Machine is capable of solving an problem. Modelled after an abstract mathematician (who has paper, pencil, and lookup rules).
- Allan Turing showed that there is a single Turing Machine that can simulate every other Turing Machine.

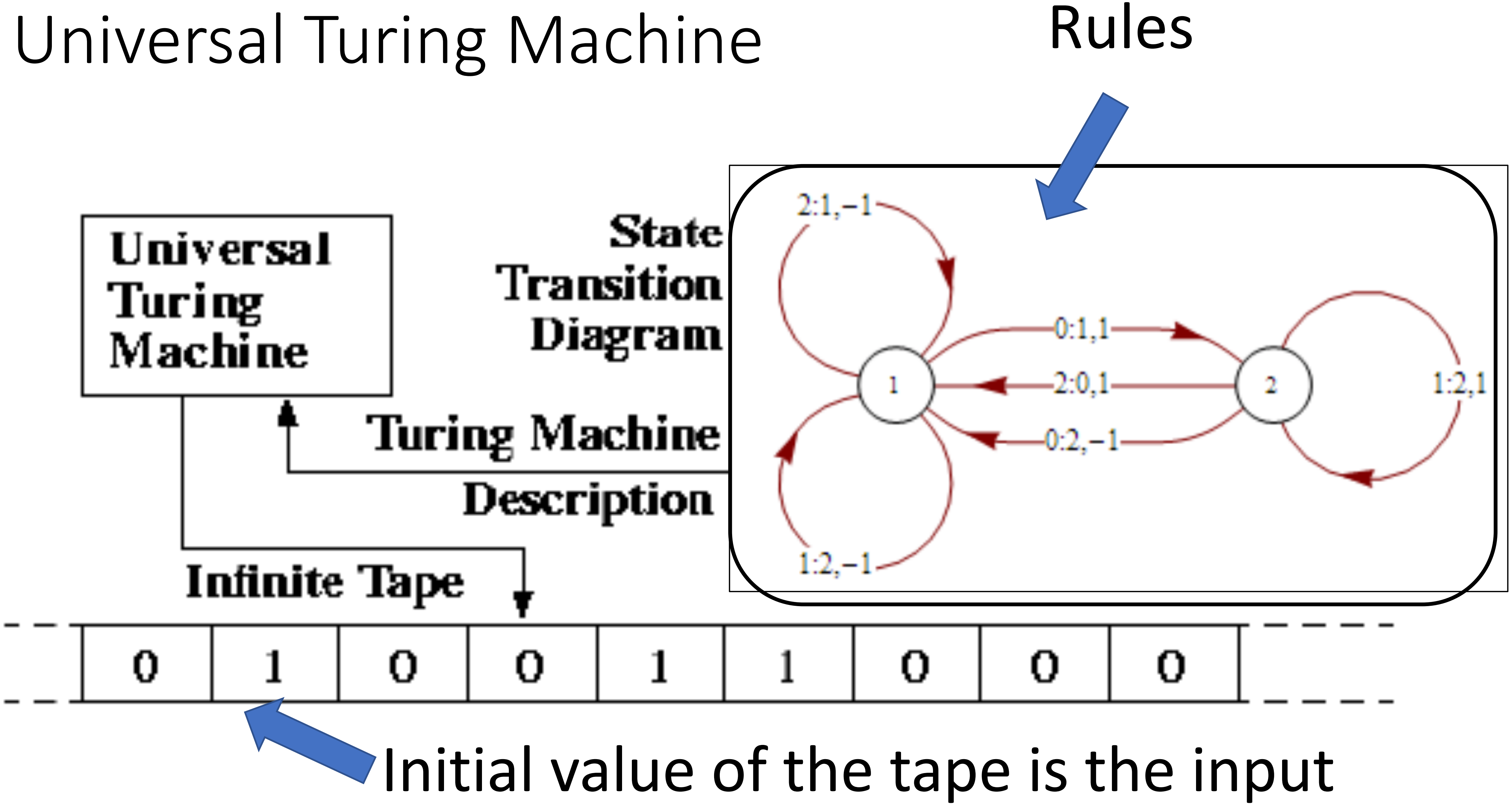
- So from simple lookup rules, pencil and paper we can generate the entire diversity of the computational universe.

- Von Neumann’s self-reproducing automata was the first universal CA.

What is a program really?

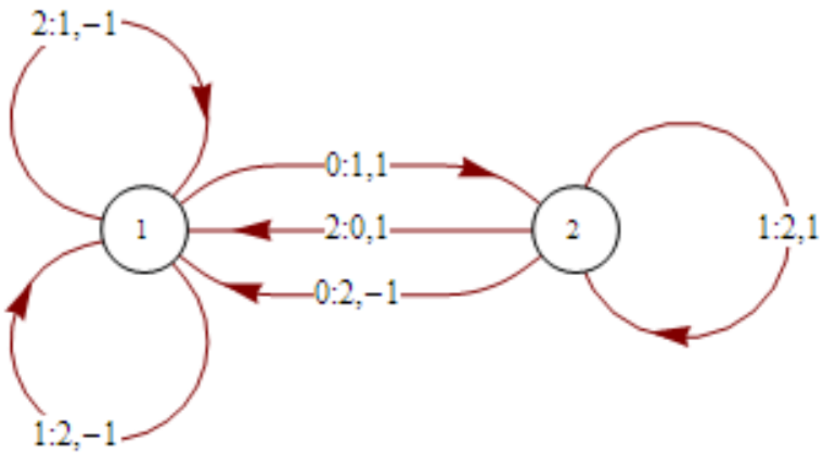
- A set of rules
- An input state
- An output

Universal Turing Machine



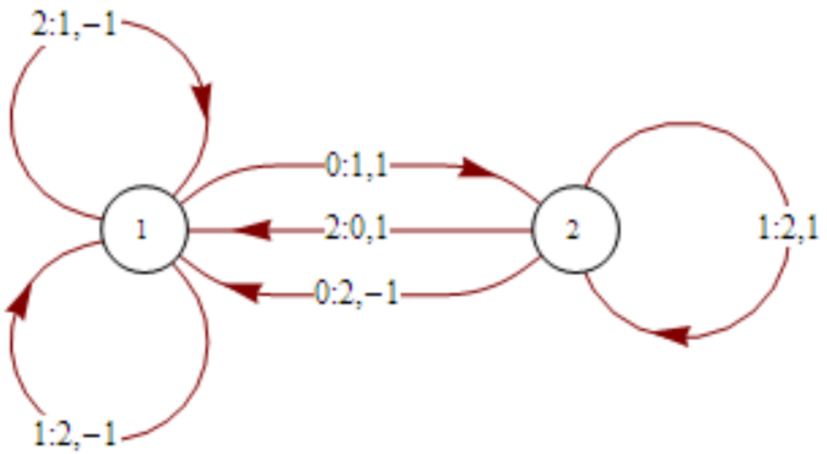
Universal Computation

- The “brain” of the mathematician in a Turing Machine is a finite state automata:



Universal Computation

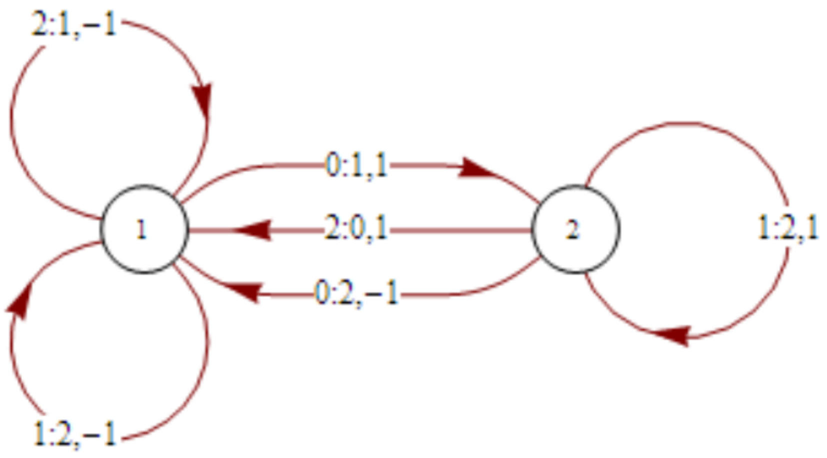
- The “brain” of the mathematician in a Turing Machine is a finite state automata (FSA):



- Wolfram showed that you can encode these FSA as rules in a cellular automata.

Universal Computation

- We are no longer thinking about elementary CAs where we just look at the neighbouring cells.



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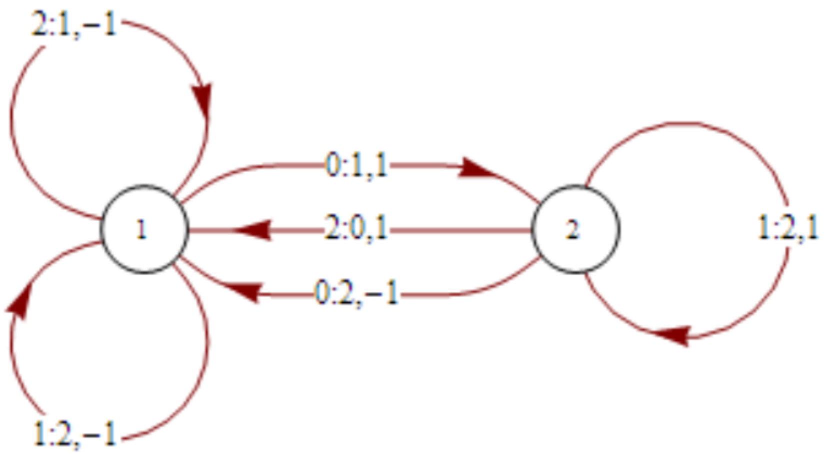
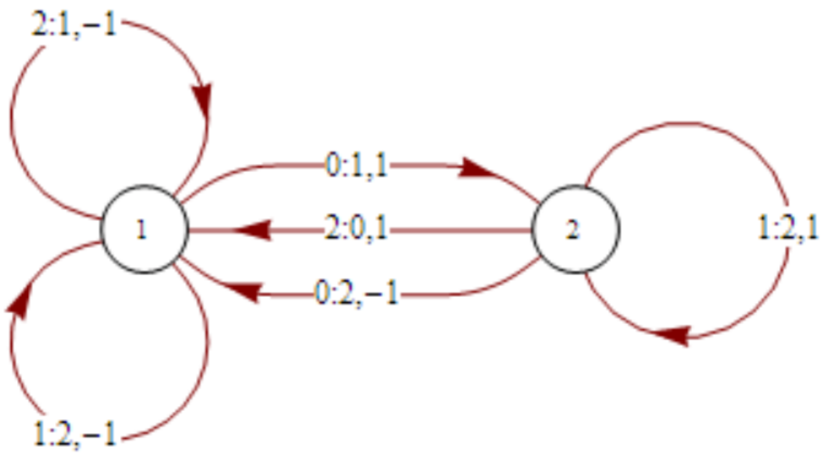


- In this example there are only 3 symbols and 2 states in the FSA. The 1 and -1 after the comma indicate the offset.

Universal Computation

- For elementary machines we saw there are 2^8 rules for 2^3 inputs.

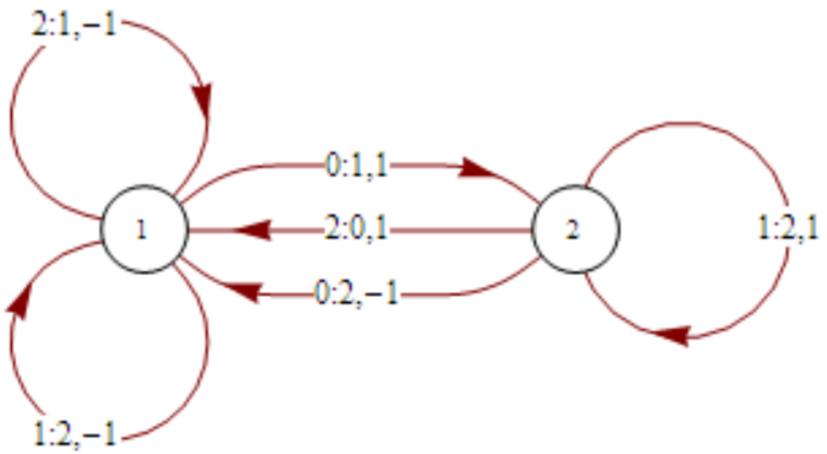


- The number of Wolfram's Turing Machine CAs is larger

$$(2|\Sigma||S|)^{2\cdot 3} = 12^6 = 2985984$$

Universal Computation

- This is machine number 596440 in Wolfram's scheme.

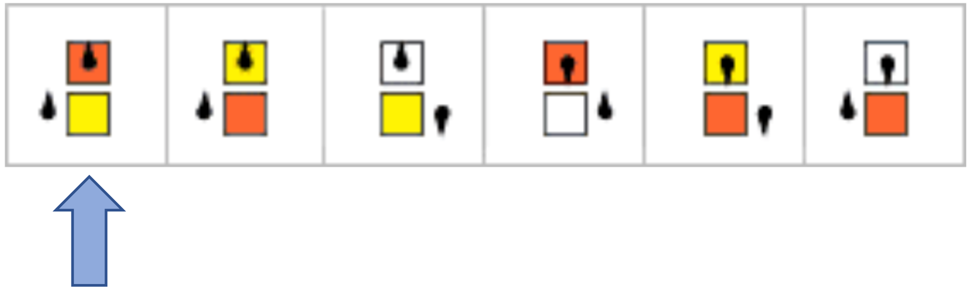


- The number of Wolfram's Turing Machine CAs is larger

$$(2|\Sigma||S|)^{|\Sigma||S|} = 12^6 = 2985984$$

Translating a Turing Machine into a CA.

	In State A	In State B
0	Print 1, Move Right, **Goto State B**	Print 2, Move Left, **Goto State A**
1	Print 2, Move Left, **Goto State A**	Print 2, Move Right, **Goto State B**
2	Print 1, Move Left, **Goto State A**	Print 0, Move Right, **Goto State A**

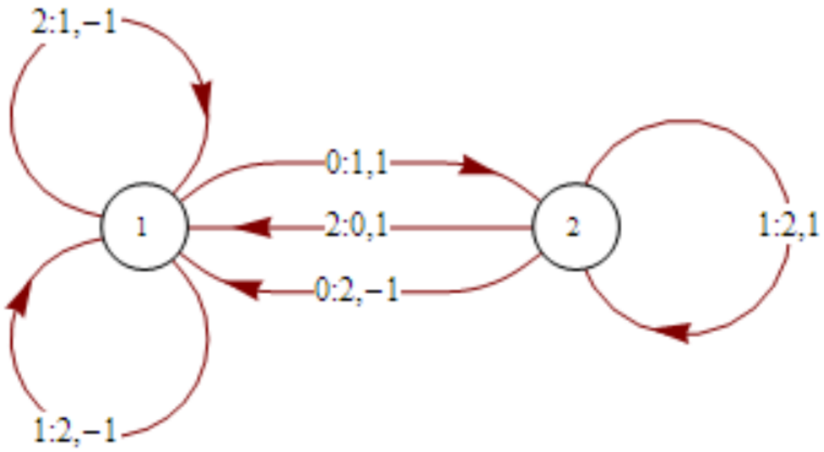


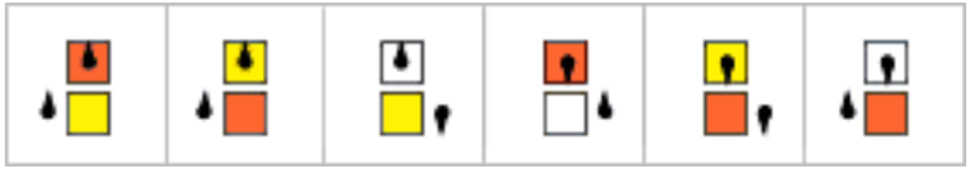
Colours = symbols on the tape
Each column is the transition from tape input to output
The droplet indicates whether the tape read head moved left or right.

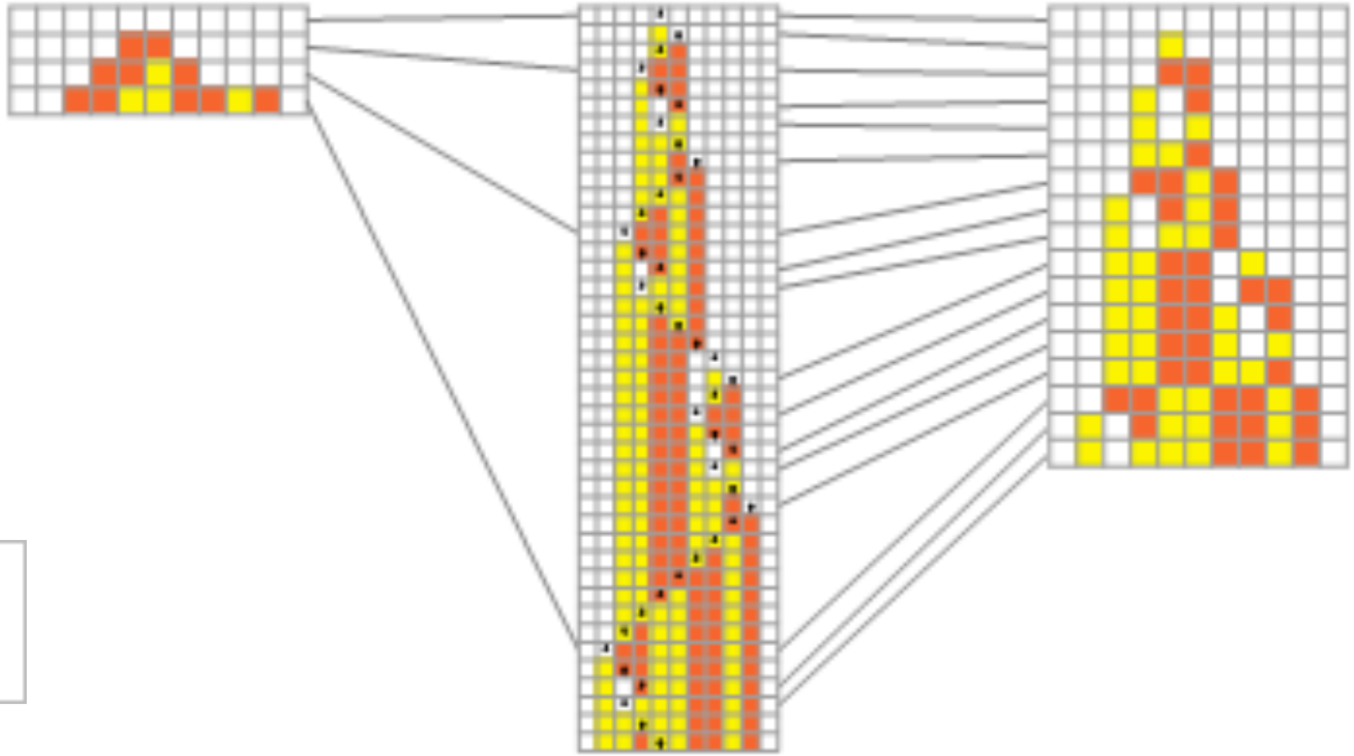
Claimed but not proven that this is the smallest possible universal Turing machine (the proof by Alex Smith is being debated)

Universal Computation

- This is machine number 596440 in Wolfram's scheme.







Cellular Automata

One-dimensional cellular automata can be universal.

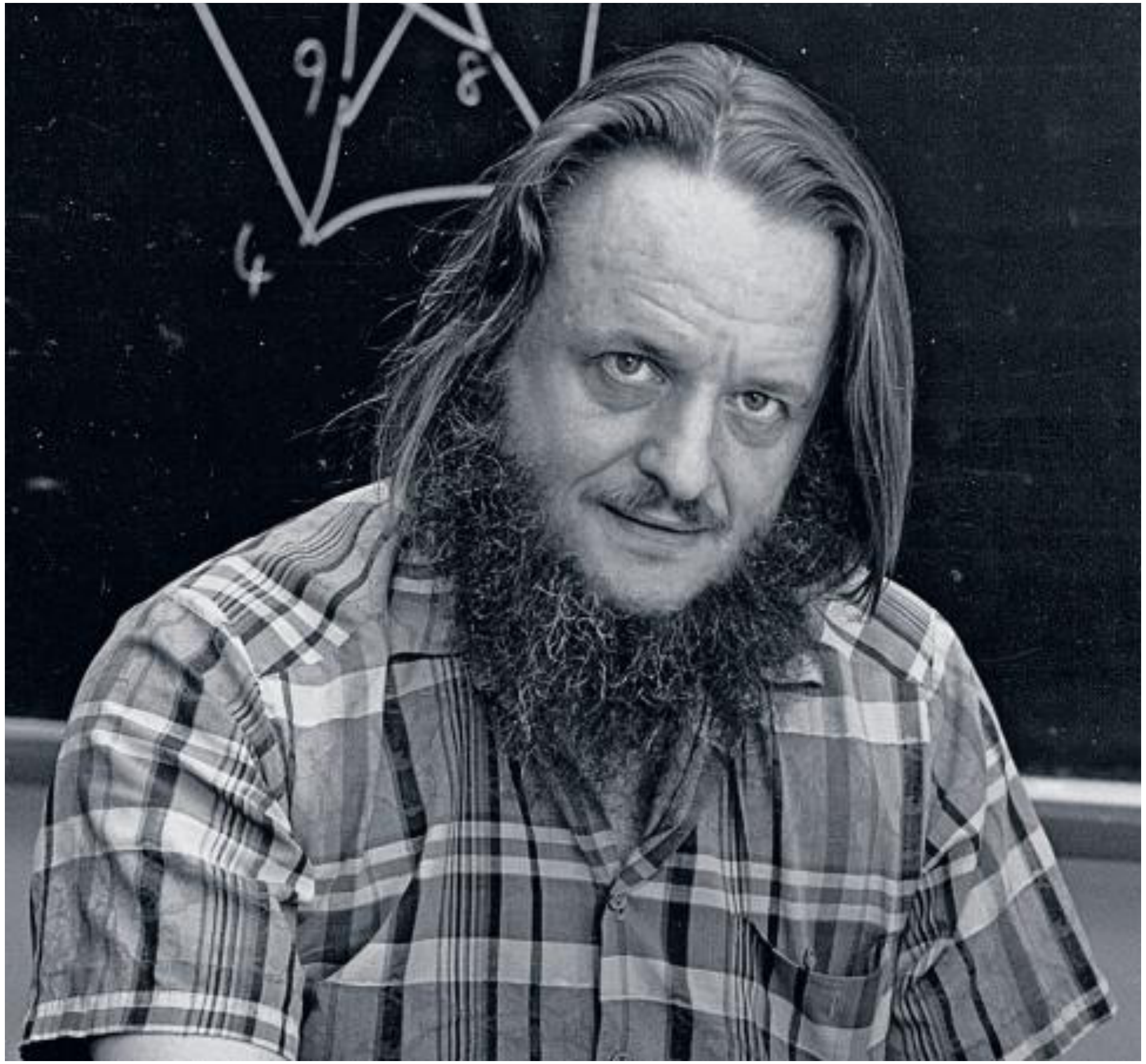
Wolfram (New Kind of Science, 2002) gave an example of a 19-color universal one-dimensional next-nearest neighbor cellular automaton in which a block of 20 cells is used to represent each single cell in the cellular automaton being emulated.

Even more amazing rule 110 elementary cellular automaton is universal.
(Cook, M. "Universality in Elementary Cellular Automata."
Complex Systems **15**, 1-40, 2004.).

2D Automata: Game of Life

- John Conway was interested in simplifying John von Neumann's rules for self-reproducing automata.

- Invented a 2D cellular automata to demonstrate this in 1970.

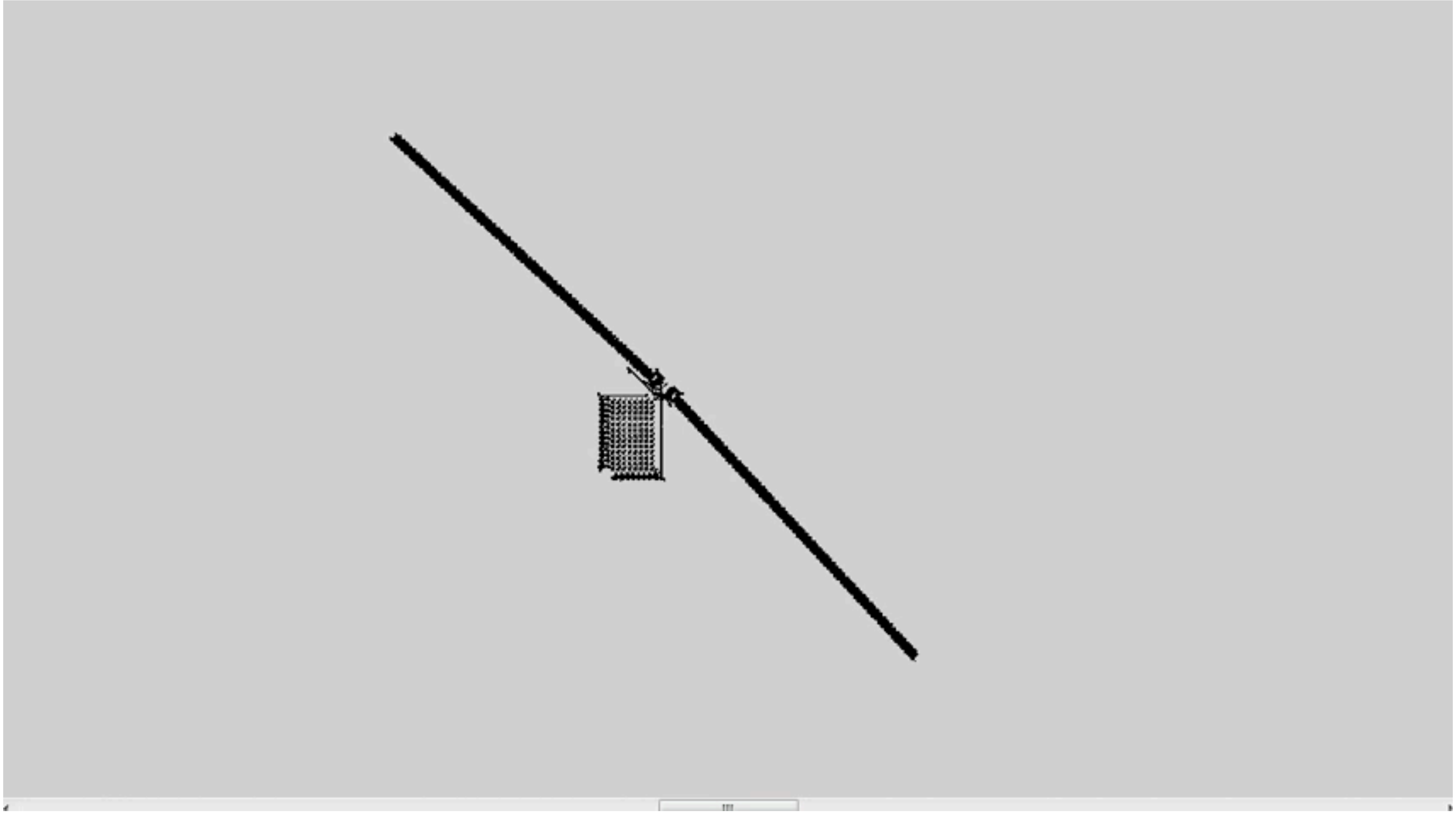


Numberphile: https://www.youtube.com/watch?v=R9Plq-D1gEk

2D Automata: Game of Life - Rules

- Any live cell with fewer than 2 live neighbours dies
- Any live cell with more than 3 live neighbours dies
- Any dead cell with exactly 3 live neighbours becomes alive
- Otherwise (2 or 3 neighbors) the cell is unchanged

Alex Bellos: https://www.youtube.com/watch?v=vGWGeund3eA



Rendell, Paul, Turing Machine Universality of the Game of Life, Springer, 2016

Models

- We have seen a lot of different models in this class being used for different things.

- But what is a model? What can they do?

Models

Can you think of a model we have seen in class that address each of the following. (Work with the person next to you)

- A model that makes *predictions* about some system
- A models used to define *computation*
- *Existence proof* models (models demonstrating the possibility of something).
- A model used to *explain* something that already happened.

Models

- `Now it would be very remarkable if any system existing in the real world could be *exactly* represented by any simple model. However, cunningly chosen parsimonious models often do provide remarkably useful approximations.´
- `For such a model there is no need to ask the question "Is the model true?". If "truth" is to be the "whole truth" the answer must be "No". The only question of interest is "Is the model illuminating and useful?”´.

Box, G. E. P. (1979), "Robustness in the strategy of scientific model building", in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics, Academic Press, pp. 201–236.

Models

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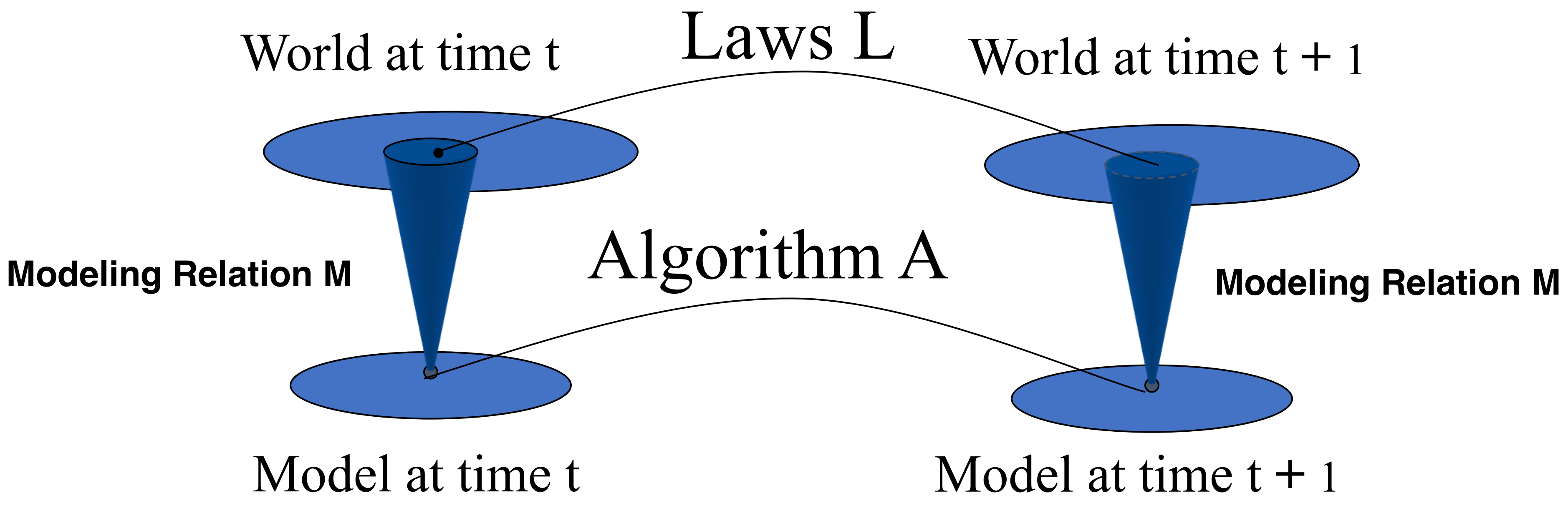
- `For such a model there is no need to ask the question "Is the model true?". If "truth" is to be the "whole truth" the answer must be "No". The only question of interest is "Is the model illuminating and useful?”´.
- “All models are wrong, some are useful.”

Box, George. E. P. (1979), "Robustness in the strategy of scientific model building", in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics, Academic Press, pp. 201–236.

Models as Homomorphic Maps Commutativity of the Diagram



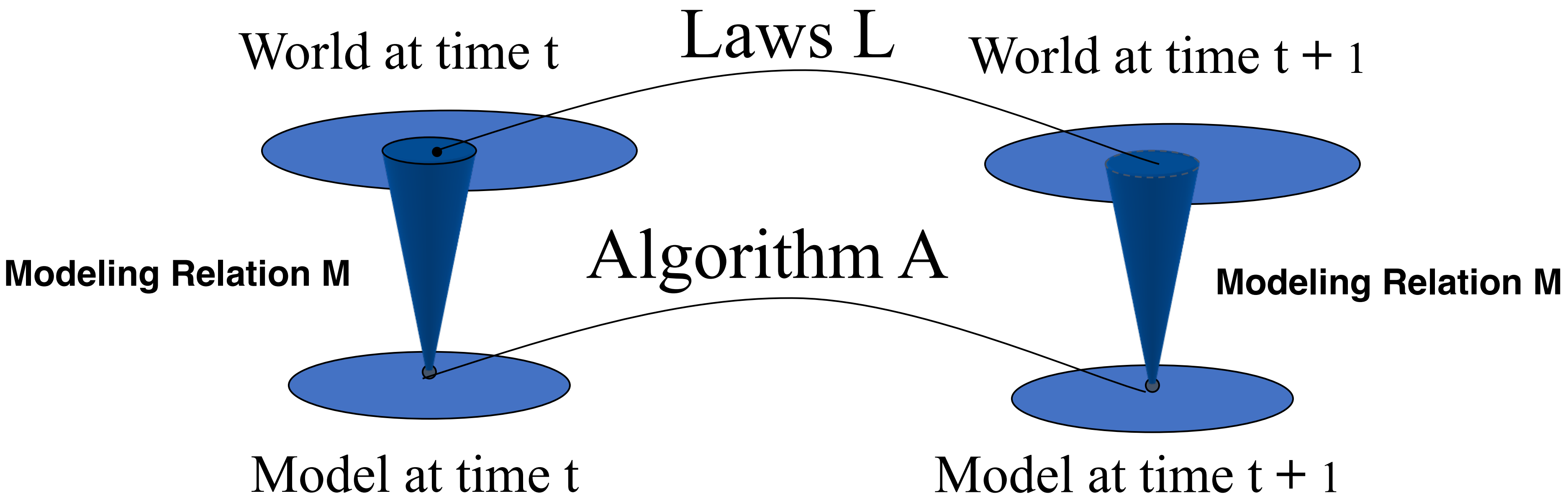
M is an equivalence relation.

Model M is valid if this is a homomorphic map:

M(L(x)) = A(M(x))

Models as Homomorphic Maps

transformation of one set into another that preserves in the second set the relations between elements of the first.



Models

- “It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.”

Attributed to Albert Einstein in “On the Method of Theoretical Physics,” the Herbert Spencer Lecture, Oxford, June 10, 1933. This is the Oxford University’ Press

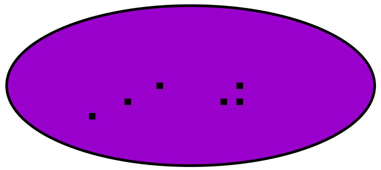
Equivalence Classes (CS261)

- Equivalence class =

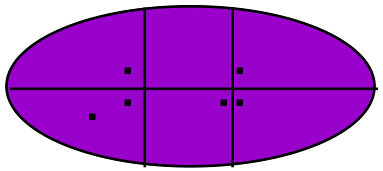
  $\{x \mid x \in R\}$ and R is an equivalence relation.

- R is an equivalence relation:
  - Reflexive: $\forall x (xRx)$
  - Symmetric: $xRy \Rightarrow yRx$
  - Transitive: $(xRy) \wedge (yRz) \Rightarrow (xRz)$

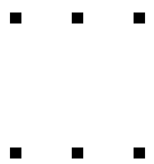
- Example: xRy <=> x and y are in the same little box.



Set of Objects



Partition set into 6 little boxes



Equivalence classes

Stephanie Forrest

Examples of Equivalence Relations

- “Is similar to" or "congruent to" on the set of all triangles.
- Logical equivalence of statements in logic.
- "Has the same image under a function" on the elements of the domain of the function.
- What’s not an equivalence relation?
  - The relation "≥" between real numbers is reflexive and transitive, but not symmetric. For example, $7 \geq 5$ does not imply that $5 \geq 7$. It is, however, a partial order.
  - The relation "is a sibling of" on the set of all human beings is not an equivalence relation.
    - Is Symmetric (if A is a sibling of B, then B is a sibling of A)
    - Not reflexive (no one is a sibling of himself),
    - Not transitive (since if A is a sibling of B, then B is a sibling of A, but A is not a sibling of A).

Stephanie Forrest

Example Homomorphism:

Multiplication of Integers

- Model all pairs of integers and their product:
  - e.g., 14792 x 4183584 = 61883574528
- Model:
  - Even X Even = Even
  - Even X Odd = Even
  - Odd X Even = Even
  - Odd X Odd = Odd

Example Homomorphism:
Multiplication of Integers

Model:
Even x Even = Even
Even x Odd = Even
Odd x Odd = Odd
Odd x Even = Even

Model relationship:

2n x 2m = 2k *R* Even x Even = Even
2n+1 x 2m+1 = 2k+1 *R* Odd x Odd = Odd
2n x 2m+1 = 2k *R* Even x Odd = Even
2n+1 x 2m = 2k+1 *R* Odd x Even = Even

M(L(x)) = M(2n x 2m = 2k) = Even x Even = Even

The relationships are preserved under our model.

Allele Dominance

Recall Gregor Mendel's experiments with peas

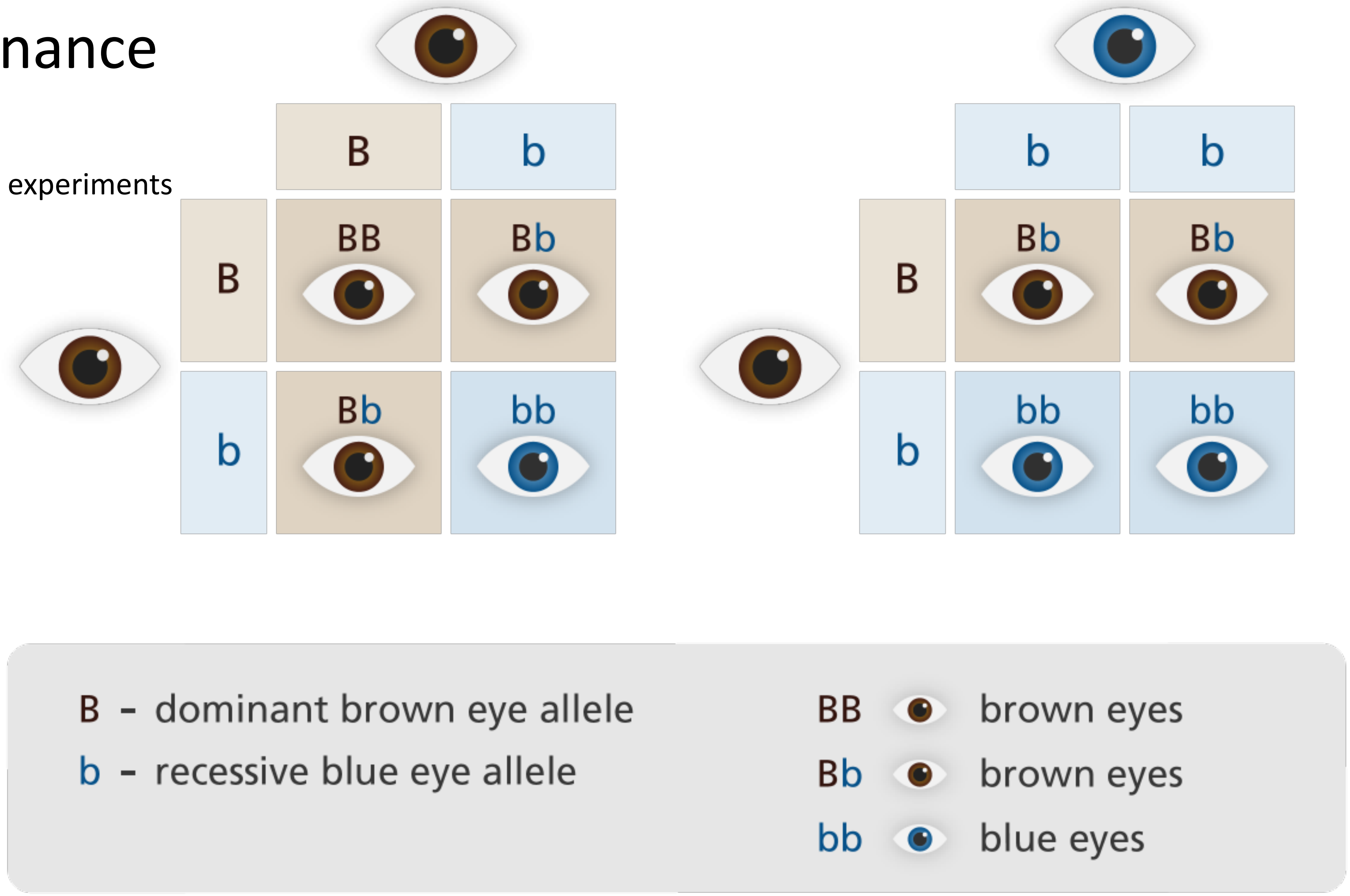
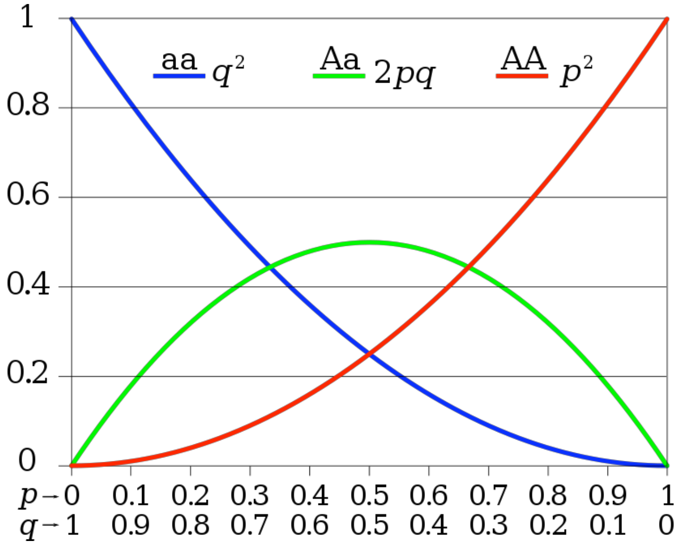


Image credit: Genome Research Limited

Example: Hardy-Weinberg Equilibrium

- Allele and genotype frequencies in a population remain constant from generation to generation in the absence of disturbing influences, such as:
  - Non-random mating, mutation, selection, limited population size, overlapping generations, random genetic drift, gene flow, and meiotic drive
- Impossible in nature. Genetic equilibrium is an ideal state that provides a baseline against which to measure change.



freq(A) = p; freq(a) = q; p + q = 1. If the population is in equilibrium, then we will have freq(AA) = p^2 for the AA homozygotes in the population, freq(aa) = q^2 for the aa homozygotes, and freq(Aa) = 2pq for the heterozygotes.

Source: Wikipedia

Stephanie Forrest

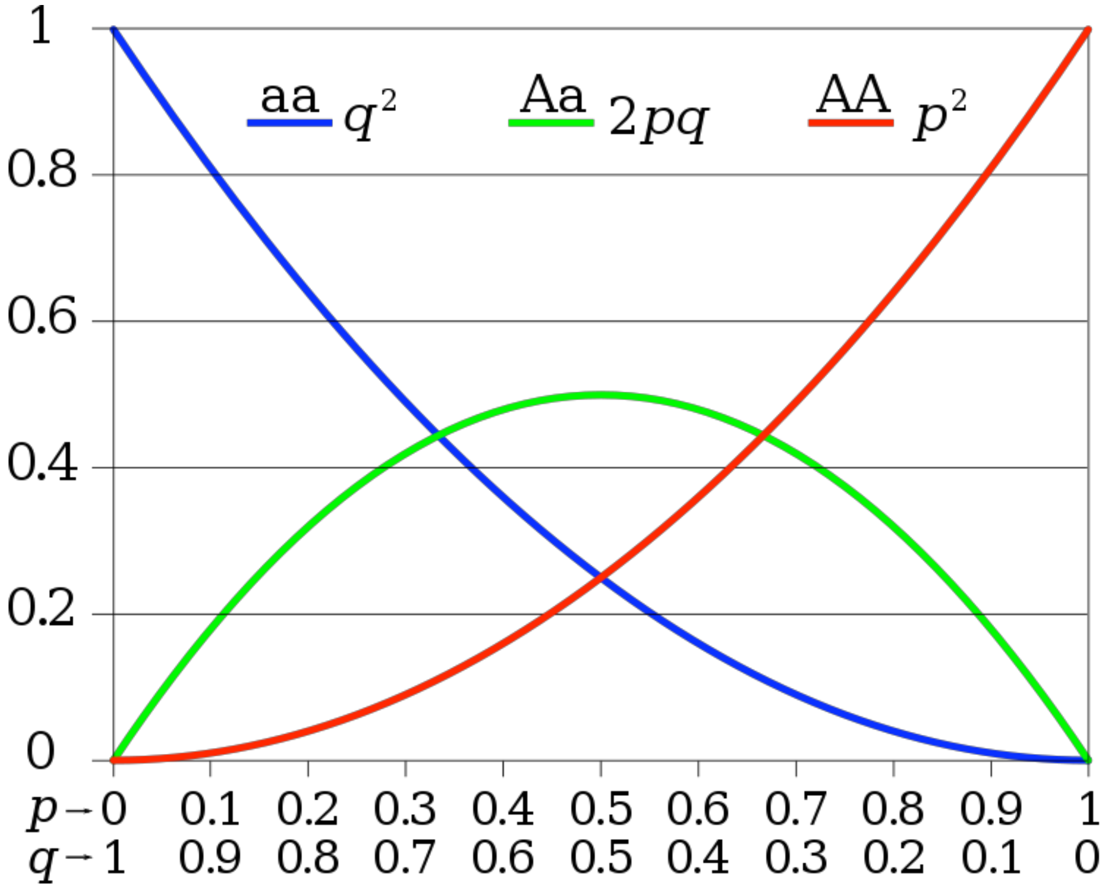
Homomorphic Model (sketch)

Hardy-Weinberg Equilibrium

- Taken from Melanie Moses (2000).
- Example CAS:
  - Theoretical population of desert shrubs, randomly distributed across a homogeneous landscape and isolated from all other populations of shrubs.
  - The shrubs have one relevant genetic variation with two values (alleles).
  - Genotype AA
    - Thrives in annual mean high temperatures T ($29 <= T <= 33$).
    - Let X = number of AA individuals.
  - Genotype Aa (and aA)
    - Thrives in annual mean high temperature $28 <= T <= 32$.
    - Let Y = number of Aa and aA individuals.
  - Genotype aa
    - Thrives where $27 <= T <= 31$.
    - Let Z = number of aa individuals.
  - Need some rules for how populations change (not shown).
- Set of states for system are all (X,Y,Z,T) s.t. $X+Y+Z=N$ for all T:
  - Fixed size population.

Hardy-Weinberg Equilibrium cont.

- Equivalence classes for this set are all X, Y, Z, T such that the ratio of X:Y:Z is constant and T is constant:
  - E.g., <100, 200, 100, 30> and <200, 400, 200, 30> are in the same class.
  - Can show that this defines an equivalence relation.
- Transition rules for model are of the form:
  - If 29 <= T <= 31, then population remains in Hardy-Wienberg equilbrium: $X=p^2$, $Y=2pq$, and $X=q^2$. (p=proportion of A's in population, q = 1-p).
  - If T=32: $X=p^2 + 0.05q^2$; $Y = 2pq + 0.05q^2$; $Z = 0.9q^2$
  - Etc.
- Now, show that this is a homomorphism.



Source: Wikipedia

Homomorphic Models Final Remarks

- What if the inside of the box is not deterministic?
  - Search for a set of inputs/outputs that take more variables into account (refine the model) and see if the result is deterministic. OR
  - Look for statistical determinacy, i.e. determinacy in the averages, as in Markov chains.
- Models themselves are seldom regarded in all their detail:
  - Only *some* aspect of the model is related to the system of study.
  - E.g., a tin mouse may be a satisfactory model of a living system, if we ignore the "tinniness" of the model and the "proteinness" of the living mouse.
  - Thus, two systems (the system of study and the model) are often related such that a homomorphism of one is isomorphic with a homorphism of the other.

Lattice Gas Models (LGCA)

- Gasses and fluids can be modelled with continuous models
- That is, we can use continuous values of pressure, temperature, and velocity

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- If we are modelling systems with very high energies (such as a nuclear explosion) we have to have a discrete model of the internal states of the atoms involved.

Lattice Gas Models

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- These systems require models of the **microscopic** behaviour

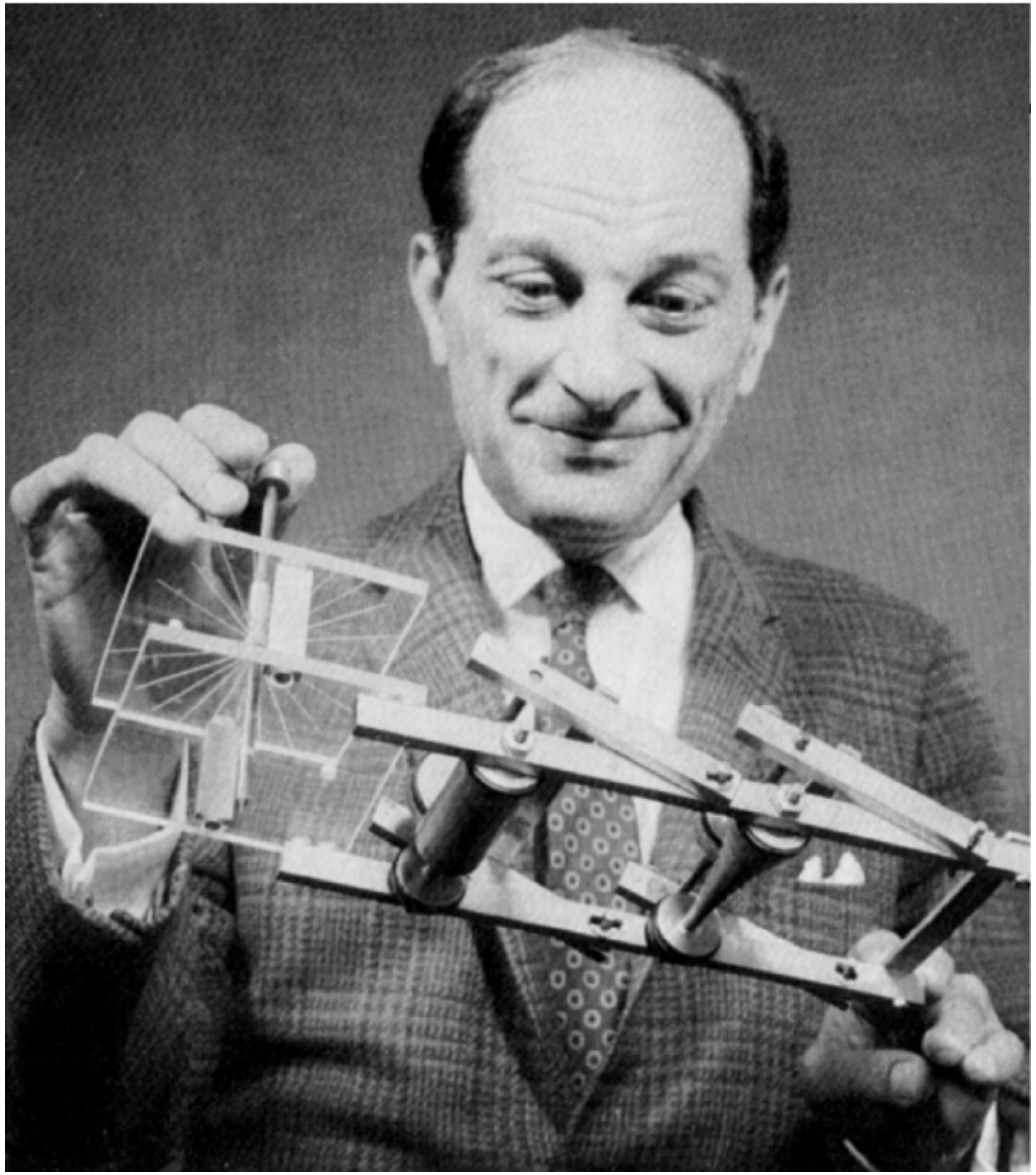
- Models that are able to describe the behaviour of the system using just pressure, velocity, and temperature are **macroscopic**.

- Of course, we could model all gasses and fluids at the microscopic level.

Lattice Gas Models

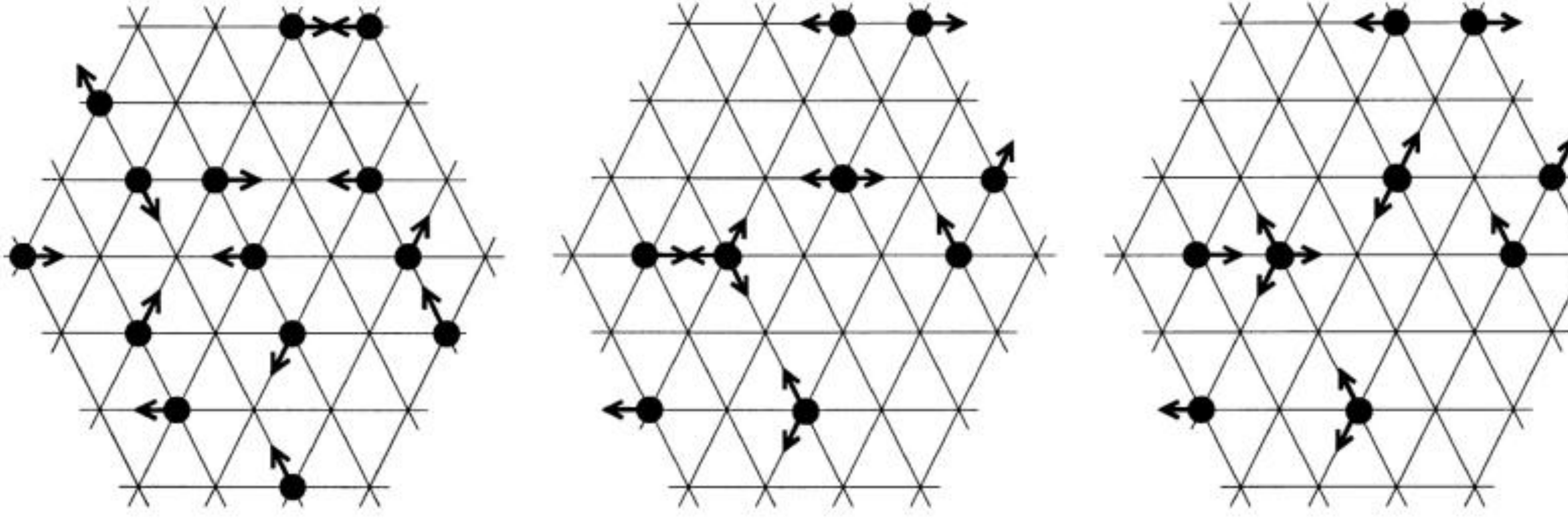
- Cellular automata are used to model molecular systems.

- The use of cellular automata to model particles such as gasses, fluids, and
- The propagation of subatomic particles was pioneered by Stanislaw Ulam and John von Neumann in the 1950s.



Stanislaw Ulam with the FERMIAC, used to model neutron transport, Los Alamos National Labs

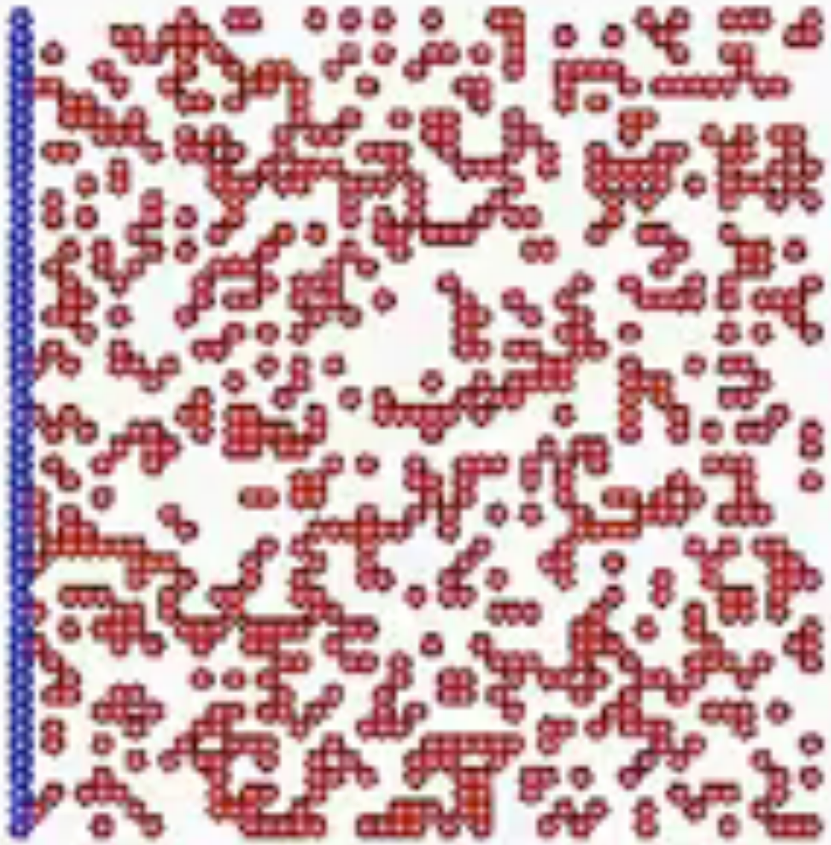
MCNP SOFTWARE QUALITY: THEN AND NOW Gregg C. Giesler, Los Alamos National Laboratory LA-UR-00-2532; 16 October 2000



(a) initial lattice

(b) propagation

(c) collision handling



Liquid Simulation on Lattice-Based Tetrahedral Meshes

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