Introduction

The following report is provided to address the issue of rabbits as an invasive species to the natural environment of Australia. Models of rabbit population dynamics have been created based on the observed population statistics in two study areas A and B. The models allow for the creation of population projections under a variety of proposed removal methods and environmental conditions. Based on these projections, recommendations for remediation and investment are offered to the Australian Rabbit Eradication Commission for their consideration.

The population dynamics for rabbits can be modeled by a logistic map [1]. The principle parameter to the logistic map is the rate R which is combination of the birth and death rates of the population. The population size is represented in the model as a percentage of the carrying capacity of environment. The dynamics of the population can be either periodic or chaotic depending on the the value of R used in the model. For low values of R, the population quickly converges over time to a fixed size or an oscillation between sizes at a detectable period. Subtly higher values of R produce results that are either chaotic or of periods so large that they cannot be determined.

Figure 1 is bifurcation diagram showing the importance of the transition from order to chaos in the logistic map. The valid population values split as the value of R increases until the bifurcations become chaotic. Whether or not the behavior of the population is chaotic greatly influences the accuracy to which predictions can be made using the model. Even a small mistake in measurement can greatly impact the population forecast when the behavior is chaotic as shown in Figure 2. This sensitivity [2] means that merely miscounting 1 rabbit in the initial population significantly restricts how far into the future that a confident estimate of the population can be given. For this reason, it is important to determine if the R value for the various rabbit populations is in or out of the chaotic regime.

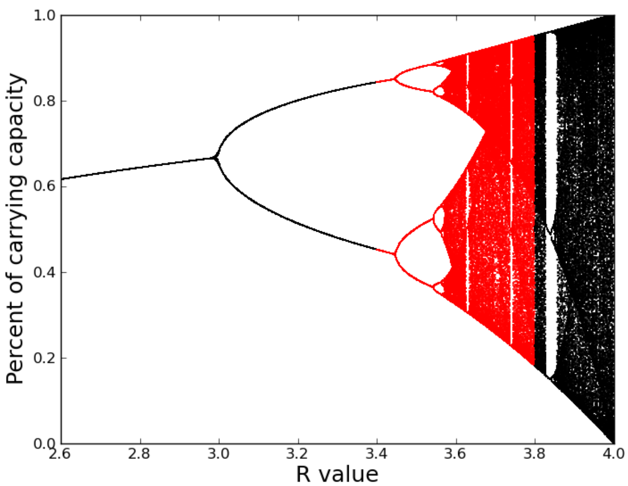

Figure 1: Bifurcation diagram with valid rates for the study in red.

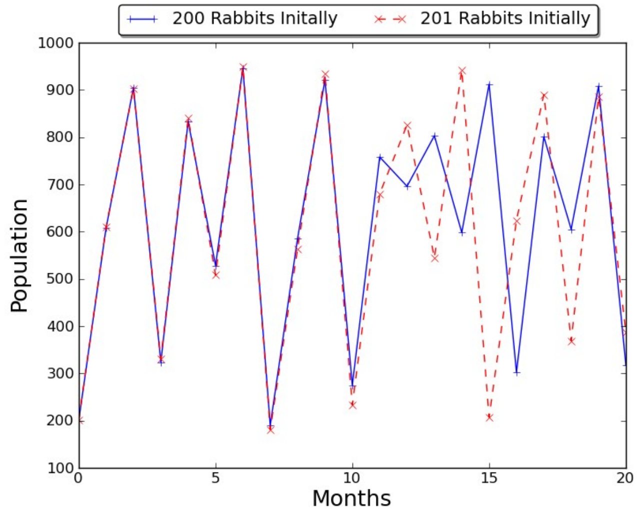

Figure 2: Example of chaotic behavior with the smallest error in initial condition.

Observations

Rabbit birth rates are known to vary between 3.8 and 4.0 rabbits per month while death rates vary between 0.2 and 0.4 rabbits per month. The value of R used to model the rabbit population may therefore vary between a minimum of 3.4 and a maximum of 3.8. Without any information about the likelihood of the various rates, the average rate can only be calculated assuming that all rates within the ranges are equally likely. Therefore the average rate of growth is 3.6 rabbits per month.

Observed rabbit population data for 100 months is available for the two study areas in question. Simple inspection can be used to determine with high confidence that the population in Area B is not chaotic. After an initial transient period, the population in Area B oscillates between four values 500, 874, 382, and 826. The situation for Area A is less clear. A long period may exist between repeated values in months 35 and 94. Even if this represents a true period it would require a minimum of 5 more years worth of rabbit population data to confirm. Even if the growth in Area A is not chaotic, it is sufficiently close to chaos that the concerns surrounding model sensitivity remain.

Estimates for the growth rate can be made for each of the study areas using the population data. For this analysis, it is sufficient to examine only the first two data points in each population series. The values are used in the equation for the logistic map which is then simply solved for R (see function `estimate_r` in the project source code). This method can be validated by comparing the output of 100 iterations of the logistic map with the observed population data. Using this method, the rates for areas A and B are 3.7 and 3.5 respectively. it is not surprising that Area A has a higher rate than Area B due to the more chaotic nature of the population dynamics in Area A. The observed population numbers compared with the logistic map using the estimated R values can be seen in Figures 3 and 4.

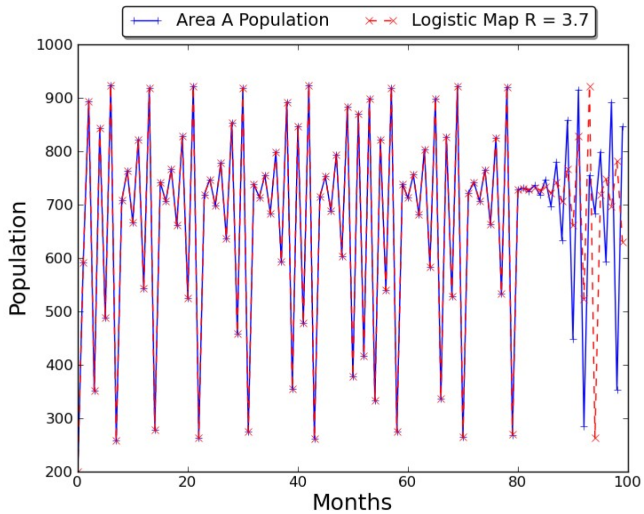

Figure 3: Comparison of observed rabbit population in Area A with the logistic map.

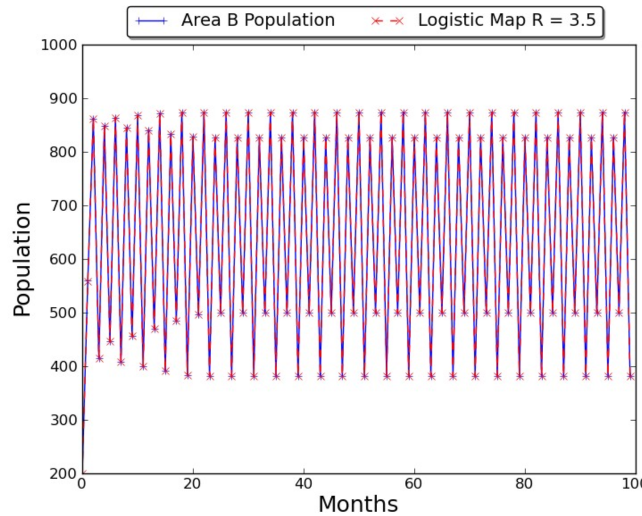

Figure 4: Comparison of observed rabbit population in Area B with the logistic map.

While rates are real valued, the rabbit population can only be represented by integers. This allows for other rates slightly higher than the ones calculated that produce accurate values for the data when rounded down to a whole rabbit. These alternative rates deviate from the observed data depending on the magnitude of the difference with the calculated rates and the number of months of observed data used for comparison. A non-inclusive upper bound can be calculated by repeating the original rate calculation with one rabbit added to the expected output of the logistic map. The bounds for Areas A and B are 3.70625 and 3.50625 respectively.

The total number of rabbit months for areas A and B are 66802 and 63960 respectively. This represents less than a 5% difference between the two areas. The absolute value of rabbit months between the two areas will depend on the particular time frame of the comparison. Both amounts are also limited by the carrying capacity of the environment which is the same in both areas. In this particular case, the area with the higher growth rate and more chaotic dynamics also has a larger number of rabbit months. This is not necessarily true universally which is an important point to consider when evaluating rabbit removal methods.

Projections

The model for the two study areas can be used to produce projections of the rabbit population. Projections begin using the last observed value of the rabbit population as the initial state in the logistic map. For the first month after the observed period (101st month) the model projects that 482 rabbits will live in Area A and 826 in Area B. For 20 months after the observed period there will be a total of 12935 rabbit months in Area A and 12918 in Area B.

Alternate projections using the upper bound of the growth rate gives total rabbit months of 13131 for Area A and 12917 for Area B. In this comparison of worst case error in the estimate of the growth rates, the population of the 20 month projection in Area A is shown to be off on the order of hundreds of rabbits while Area B only suffers by single digit errors. Confidence in the accuracy of the projections is therefore much greater for Area B than it is for Area A. This is not surprising given the observed chaotic nature of Area A.

Three rabbit removal methods are available to the commission. The methods can be modeled using slight modifications to the logistic map and projections can be used to evaluate their comparative effectiveness. Poison is capable of reducing the initial rabbit population by 98% which is modeled by simply reducing accordingly the initial population value as input to the logistic map. Hunting is capable of removing 120 rabbits per month from the population which is modeled by subtracting the same proportion of removed rabbits from each step in the logistic map. Finally, an intentionally introduced disease is capable of reducing the rabbit population by 20% each month. The disease method is modeled by multiplying the output of the logistic map by 0.8 at each step. For exact implementation details, please refer to the project source functions `lmap`, `lmap_poision`, `lmap_hunt`, and `lmap_disease`.

Projections for the total rabbits months removed are shown in Table 1 for each area and removal method. The duration of the projection is 100 months with an initial rabbit population of 200. The number of rabbit months removed is the difference between a projection with no removal method and a projection with the removal method. Note that the difference is not calculated using the directly observed values to avoid making the presence of the model a factor in the calculation.

Removal Method	Area A	Area B
Poison	3497	2122
Hunting	4169	421
Disease	1692	550

Table 1: Removed rabbit months per area for each removal method.

The level of uncertainty regarding these projections is again dependent upon the area in question. Using the same method for measuring uncertainty in the standard projection, the error for Area A can be on the order of hundreds of rabbits while Area B can be on the order of tens of rabbits.

The behavior of the logistic map can involve a transient phase where any periodic behavior of the population may not be seen until after the first few iterations. To take this into account, the projections for each removal method were repeated including an extra 10 months and with the statistics for the first 10 months ignored. The results of this analysis are shown in Table 2 and can be seen in comparison to the original projections in Figure 5.

Removal Method	Area A	Area B
Poison	1790	368
Hunting	3899	306
Disease	1603	444

Table 2: Removed rabbit months per area for each removal method without the transient phase.

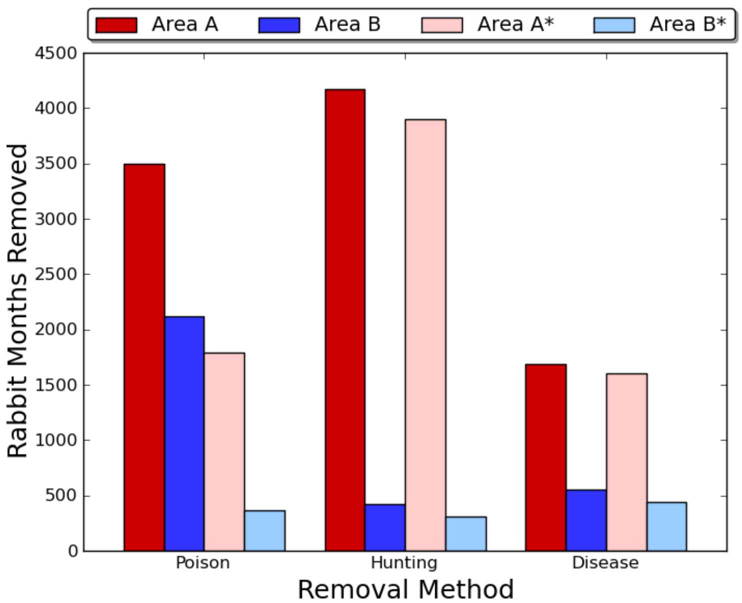

Figure 5: Comparison of removal method per area.
* (without transient phase)

Taking into account the transient phase has the most significant impact on the projections for the poison removal method. The projections differ greatly for poison because the effect is most prominent during the transient phase when the rabbit population experiences near elimination. Although poison is very effective in the short term, these results show clearly how the poison method suffers when projecting far enough into the future. The difference in the effectiveness of poison when considering the transient phase brings it nearly on par with the effectiveness of the disease method.

The hunting method is projected to remove 4205 rabbits from both areas combined which is nearly twice as much as the other two methods. Therefore hunting is the method that is recommended to the commission for rabbit removal **given the current state of the environment**. Any error produced by the uncertainty in the projections changes them proportionally amongst the removal methods. Additionally, minor improvements to the other methods do not make them competitive with hunting. The hunting recommendation is therefore robust under considerations of any expected error in the assumptions that govern the projections.

Consideration of Climate Change

Rabbit growth rates are affected by the climate in their environment. Where there is less than 40cm of rainfall, rabbit birth rates decrease by 10% while rabbit death rates increase by 10%. It is somewhat difficult to take this difference into account since the calculated rates used in the prior models are not separated into birth and death components. Nevertheless, the effect of climate change on the growth rate of the rabbits can be represented by a range of values when taking the range of valid birth and death rates into account.

For example, the rates for Area A can be between 4.0 to 3.9 for birth rate and 0.2 to 0.3 for death rate. Similarly, the rates for Area B can be between 3.9 to 3.7 for birth rate and 0.2 to 0.4 for death rate. While the effects of climate change can be used to recalculate rates at any point along the continuum of separated rate values, the practical effect on the overall results is small. Therefore for this report the largest rate values within the range have been chosen to illustrate the maximal effect that climate change can have on the populations. The climate change affected rates used in the subsequent analysis for areas A and B are 3.27 and 3.07 respectively.

When the new rates are used to repeat the removal analysis, there is a dramatic change in results. Figures 6 and 7 show the population projections for each area using the new rates. The most important point to emphasize is the periodic nature of the projections in both cases. The lack of chaos provides an opportunity to be much more certain about the projections than was warranted by the previous rates.

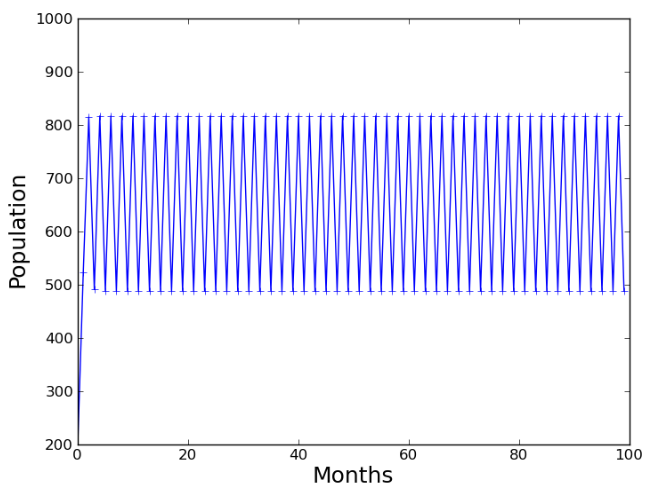

Figure 6: Projected rabbit population per month for Area A under climate change.

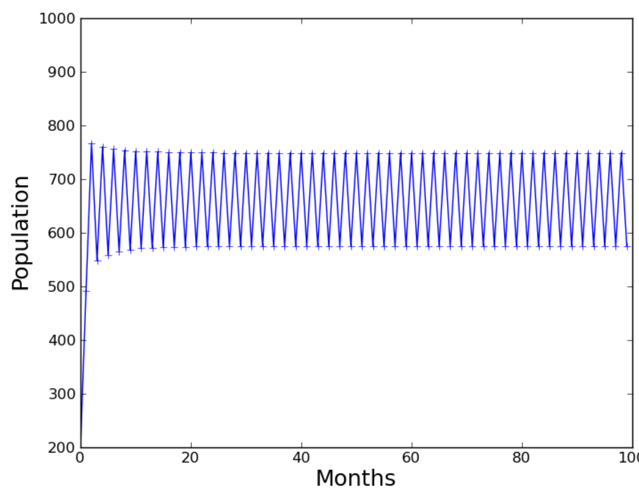

Figure 7: Projected rabbit population per month for Area B under climate change.

The rabbit removal projections for the various removal methods are shown in Table 3 and Figure 8. The disease method is now shown to be the most effective by removing a projected 9853 rabbit months. The disease method is preferable in both areas with or without taking into account the transient effects of the model.

Removal Method	Area A	Area B
Poison	321	-126
Hunting	1659	4823
Disease	3394	6459

Table 3: Removed rabbit months per area for each removal method without the transient phase with consideration for climate change.

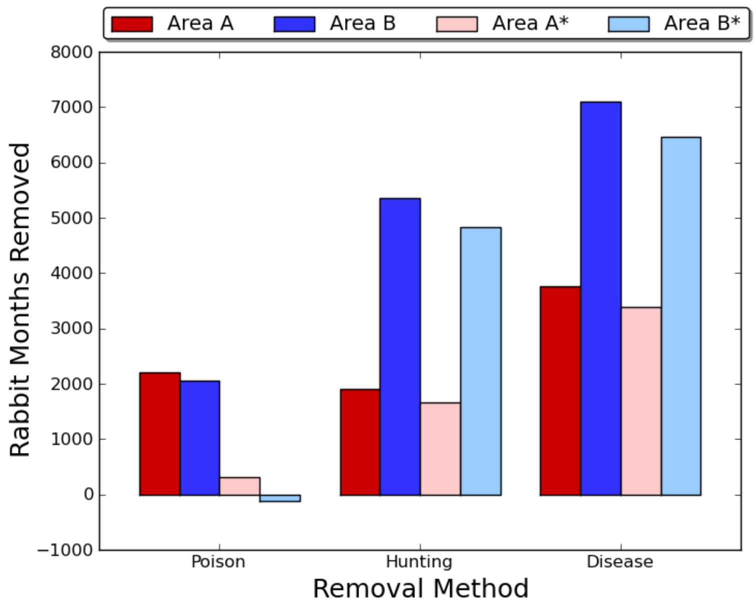

Figure 8: Comparison of removal method per area. under climate change conditions. * (without transient phase)

Failure to pair the correct removal method with the climate conditions incurs a penalty in terms of increased rabbit months. If the disease method is used in the absence of climate change then the potential to remove 2158 rabbit months is lost. Alternatively, if the hunting method is used in the presence of climate change then potential to remove 3371 rabbit months is lost.

Analysis by climatologists suggests a 75% chance that climate change will occur. The information given by these findings is represented by the entropy of the climate change random variable. This amounts to `-0.75 log(0.75) - 0.25 log(0.25) = 0.81` bits of information. This large amount of average information suggests that the risk in basing a decision of removal strategy on the climatologist's results is low.

Given both that the chances of climate change are great and that the penalty for choosing the wrong method in the presence of climate change is the most expensive, it is recommended that the disease method be used by the commission. Additional resources, if available, should be spent on improving the disease method in anticipation of climate change or as a contingency to the lost removal potential should climate change fail to occur.

Summary

This study is offered to the Australian Rabbit Eradication Commission in order to assist in decisions regarding rabbit removal methods. The most effective removal strategy depends greatly on the chance that climate change will impact the areas where the removal methods will be used. The high probability of climate change suggests that the disease removal method is the best choice for rabbit eradication.

[1] Wikipedia, Logistic Map, http://en.wikipedia.org/wiki/Logistic_map, retrieved on 2-12-2012
[2] Flake, Computational Beauty of Nature, MIT press 1998 ch 10 page 155