Introduction

This report examines whether poison, hunting, or a human-introduced disease would kill the most rabbits in two locations, “Area A” and “Area B.” The report also considers whether possible changes in annual rainfall within the next decade could affect the effectiveness of each removal method. To predict changes in each rabbit population, the report uses the logistic map.

The equation for the logistic map takes the current number of rabbits, x_t, and returns the number of rabbits that will be in an area the next month, x_{t+1}. The logistic map also takes into account an area’s birth rate, the probability overcrowding will cause a death in the population (death rate), and the area’s limit on population size (carrying capacity). Birth and death rates appear in the equation as *R,* or the difference between the birth rate and the death rate.

The equation for the logistic map is: $x_{t+1} = x_t * R * (1 - x_t)$.

According to [1], the outputs of the logistic map cycle between a set of fixed values for low values of *R*. Initially *R* will produce a cycle, or period, of 1. The size of the period doubles with increasing frequency to 2 values, and then 4 values, and so on, as *R* increases. When *R* equals around 3.57, the logistic map achieves an infinite period. The logistic map produces “chaotic” results for these higher values of *R*. Fig. 1, a “bifurcation graph,” shows how the period doubles as R increases.

Consequently, [1] states that more uncertainty will surround predictions using high values of *R.* For low values of *R,* the logistic map will always settle on the same set of values regardless of the size of the initial population (often designated as x_0). On the other hand, long-term predictions using high values of *R* will produce different results for very similar initial populations.

The logistic model cannot truly report conditions in the real world with precision. As a result, small unavoidable errors in the calculations and the initial populations may affect the accuracy of the prediction about each removal method’s effectiveness.

Methods

The experiments use the logistic model to examine how different removal methods will affect the rabbit populations of Area A and Area B. Climatic conditions may also affect the rabbits’ reproductive rates, so the last two experiments take into account predictions that climatologists have made about rainfall in the upcoming year.

Bifurcation Diagram

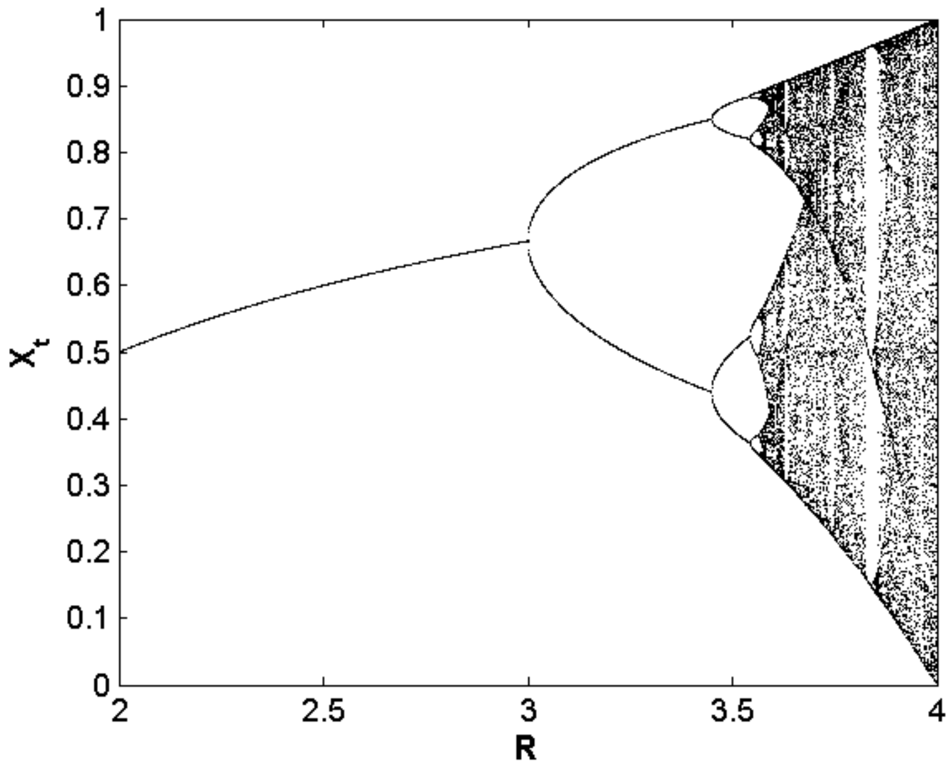

Fig. 1: *For low values of R, the logistic map has a predictable period, or set of values that it will eventually cycle between after enough iterations. The period doubles* as R *increases until it becomes infinite.*

Barring the experiments that take rainfall into account, the experiments use a minimum *R,* maximum *R,* and estimated *R* for each area. Each area has a unique range for *R* that derives from its population data.

All of the experiments and predictions using the logistic take into account that the carrying capacity of each area is 1000 rabbits.

Analysis of Population Data

Estimates for the reproductive rate, *R*, of each area must fit within the range for *R* of Area A and Area B. Birth rates for the areas span between 3.8 and 4.0 rabbits per month, while death rates span between .2 and .4 rabbits per month. These values establish a general range for *R*..

The function *analyzeRabbitData* finds the minimum *R,* maximum *R,* and estimated *R* for each area. In each area, *R* varies from month to month. *analyzeRabbitData* finds *R* for each month by using the logistic model (solved for *R*) and estimates an *R* for each area by averaging these *R*s.

The function *estimateRabbits* (see the file getMinMaxEst.m) uses the logistic model to estimate the number of rabbits that will be in Area A and Area B after 101 months and 120 months. The predictions consider the minimum *R,* the maximum *R,* and the estimated *R* for each area.

Code in "runfunctions.m" also sums together the 20 months of predictions that use the estimated *R* to give the number of estimated rabbit months for each area.

Experiments

The experiments model three different removal methods: poisoning 98% of the initial rabbit population, hunting 120 rabbits at the start of each month, and introducing a disease that kills off 20% of the rabbit population each month. Every experiment runs for 100 months.

These experiments occur in two rounds. Each round includes six tests: one for each area's minimum *R*, one for each area's maximum *R*, and one for each area's estimated *R*. The first round includes the first ten months of the 100 months, the transient phase, while the second round ignores the transient phase. As a result, each round of experiments draws on different values for the initial populations (values for x_0) from the data on Area A and Area B. The first round uses the initial populations from each area. The second round uses the numbers from month eleven, which vary for each area.

The function *estimateRabbits* performs most of the calculations for the experiments. For the poison experiments, *estimateRabbits* uses the basic logistic model. For the hunting experiments, *estimateRabbits* subtracts .12 from the initial population before calculating and storing x_1 and continues with this pattern for all subsequent months. For the disease experiments, *estimateRabbits* decreases each new population (x_{t+1}) by 20% before recording the value.

Results for each experiment are in the form of "rabbit months." To get the total rabbit months for an experiment, code in the file "runfunctions.m" adds together the estimated rabbit population for each month.

Climatologists' Predictions

Potential changes in rainfall would reduce rabbit birthrates by 10% and increase rabbit death rates by 10%. For the experiments testing the effects of decreased rainfall, the minimum *R* and maximum *R* come from the range for *R* that the given birth and death rates would establish if they changed to reflect a decrease in rainfall. The experiments also use the estimated *R*s for Areas A and B, which have decreased by 10% to reflect the shifts in birth and death rates and to fit within the new general range for *R*. To run the experiments, estimateRabbits() uses the same values as it used for the experiments that included the transient phase.

Results

The results contain estimates for *R* in each area, predictions for the rabbit populations in each area for the next twenty months, predictions for the effectiveness of each removal method under current climate conditions, and predictions for the effectiveness of each removal method under possible future climate conditions.

Estimates for *R*

Estimates for the *R* of Area A and Area B must fit within the range given by the birth rates and death rates for both areas: 3.8-4.0 and .2-.4. **The general inclusive range for *R* is 3.4-3.8. The mean *R* is 3.6.**

The minimum, maximum, and estimated *Rs* for Area A and Area B fit within the general range for *R*. Area A's has a minimum *R* of 3.65, a maximum *R* of 3.71, and an estimated *R* of 3.69. Area B has a minimum *R* of 3.47, a maximum *R* of 3.50, and an estimated *R* of 3.49.

Rabbit Population Dynamics

Area A's rabbit population dynamics are somewhat chaotic, while Area B's rabbit population dynamics are predictable. Area A's range for *R* falls within the area of the bifurcation graph where period sizes become hard to distinguish for each value of *R* (Fig. 1). **Area A's rabbit population dynamics are more chaotic than Area B's since Area A has a higher range for *R* than B. Area B's lower range for *R* will allow for good predictions of rabbit population size whereas models of Area A's rabbit population will be much more sensitive** to small changes in *R* and the initial population value, x_0.

The graphs of the data on Area A and Area B (Fig. 2) also reflect each area's range for *R*. Although Area A's graph seems to begin a pattern, this pattern disappears as the months progress. On the other hand, Area B's graph shows a period of 4.

Area A and Area B may have similar rabbit numbers (66802 and 63960) in part because of the carrying capacity, 1000. The carrying capacity places limits on how many rabbits can coexist within an area. Fig. 2 shows that Area A's rabbit population exhibits a trend of hitting close to 1000 and then suddenly plummeting down to around 300 the next month. **In Area A's case, the rabbits' higher reproductive rate actually contributes to a lower total number of rabbits for all 100 months.**[1]

[1] Note: Area B has fewer total rabbit months than Area A. I have not directly answered the question "Explain why the plot with a higher growth rate has fewer total rabbit months?"

Graphs of Data for Area A and Area B

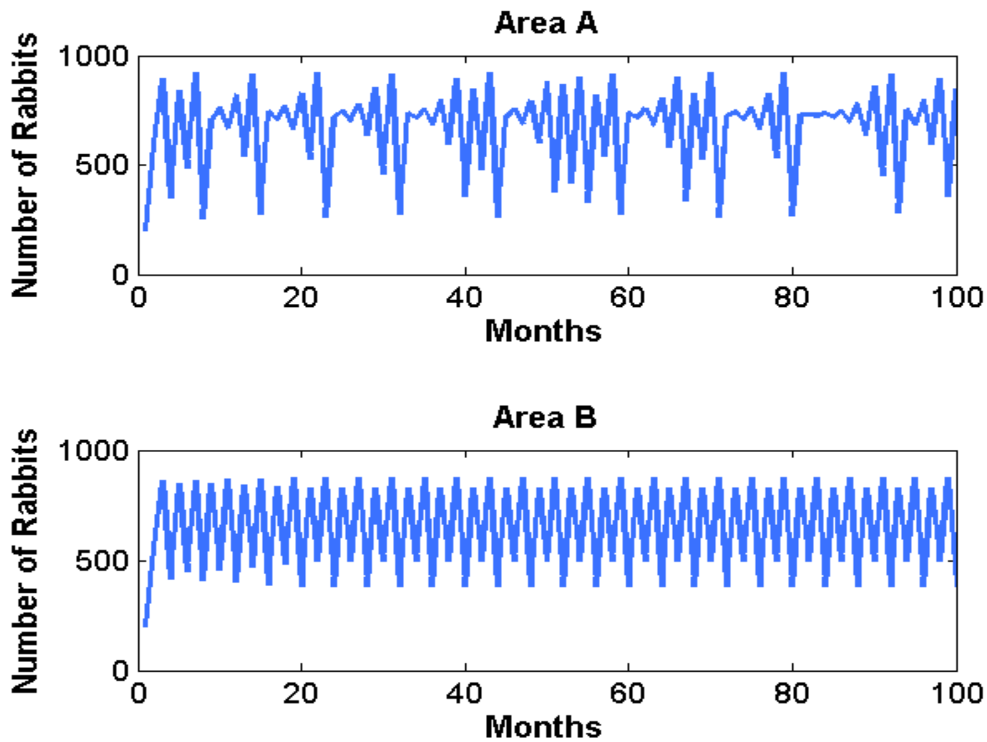

Fig. 2: *These graphs show the data on Area A and Area B. Each graph covers 100 months.*

Predictions for the Next Twenty Months

The models for Area A and Area B predict these numbers for the next 20 months:

- **Area A will have 481 rabbits after 101 months. After 120 months, Area A will have an estimated 278 rabbits. Overall, Area A will have 12994 rabbit months.**
- **Area B will have an estimated 824 rabbits after 101 months and 389 rabbits after 120 months. Overall, Area B will have 12927 rabbit months.**

Predictions for month 101 will more likely be accurate than predictions for the month 120. Chaos makes predictions further in time hard to make. **Predictions for month 101 using the estimated *R* also seem consistent with predictions using the minimum *R* and maximum *R*.** The logistic map calculates 476 rabbits and 483 rabbits (estimated *R*: 481) using Area A's minimum and maximum *R*. Predictions for Area B's minimum and maximum *R* are 819 and 826 rabbits (estimated *R*: 824).

On the other hand, predictions for month 120 may only be viable for Area B. Predictions using Area B's minimum and maximum *R* are 403 and 383 rabbits (estimated *R*: 389). Predictions using Area A's minimum and maximum *R* are 912 and 820 (estimated *R*: 278). The predictions using the minimum *R* and maximum *R* may not set a range for the predictions using the estimated *R*. **However, the numbers should be close to show that small variations in the value of *R* will only mildly affect the predictions.**

The predictions for rabbit months may be viable for both areas. The other predictions for Area B seem consistent, so the prediction for Area B's number of rabbit months is likely. The prediction for Area A's number of rabbit months will be less likely than the prediction for Area B. However, the predictions for Area A at month 120 echo the trend that appears in Fig. 2. For each *R,* Area A's rabbit population may frequently reach numbers near 1000 and plummet the next month. This trend moderates the total number of rabbit months in Area A and may increase the accuracy of predictions about Area A.

Removal Methods

Table 1 shows the estimated number of rabbit months for each area after humans have applied poison, hunted rabbits, introduced disease a disease into each area. The results in Table 1 come from experiments that include the first ten months, or the transient phase, and span 100 months. Based on the estimates from these experiments and the number of rabbit months given for each area in the data, the experiments would increase or decrease rabbit populations by the following amounts of rabbits over a span of 100 months:

- **Poison would remove 1687 rabbits from Area A and 1260 rabbits from Area B.**
- **Hunting would increase the number rabbits in Area A by 8119 and the number of rabbits in Area B by 11610.**
- **Disease would remove 896 rabbits from Area A and increase the number of rabbits in Area B by 58.**

Each estimate seems fairly consistent with the result for each minimum and maximum *R*. Due Area A's more chaotic population dynamics, the experiments will probably produce more inaccurate results for Area A and Area B.

The experiments yield different results if they ignore the transient phase. Fig. 3 shows that the higher results for the poison and hunting experiments indicate that although poison would still prove effective, it would kill fewer rabbits than the previous experiments predicted. Fig. 3 also shows that unlike in the previous experiments, disease would bring down the rabbit populations of both Area A and Area B. Disease seemed only to initially prove effective in Area A.

Based on the numbers, poison seems like the best option. Each set of experiments predicts that poison would kill the most rabbits in both areas. However, the second set of experiments, which ignores the transient phase, predicts that poison remove would fewer rabbits in both areas than the first set of experiments (947 in Area A and 282 in Area B vs. 1687 in Area A and 1260 in Area B).

Rabbit Months for Each Removal Technique			
Removal Method		**Area A**	**Area B**
Poison	Estimate	65115	62700
	Minimum R	63462	62463
	Maximum R	64431	62751
Hunting	Estimate	74921	75570
	Minimum R	75130	75610
	Maximum R	74805	75585
Disease	Estimate	65906	64018
	Minimum R	65554	63816
	Maximum R	66061	64116

Table 1: *Table 1 shows the results for the experiments measuring the effects of each removal method on rabbit populations in Area A and Area B. These results include the first ten months, the transient phase. Results for each experiment appear in rabbit months, the sum of the monthly rabbit populations over the span of the experiment.*

Area A's and Area B's rabbit populations would react differently to the more rigorous removal methods than to poison. **As a removal method, poison would allow natural growth patterns to resume after the first day. The more rigorous methods (particularly hunting) might allow each area to sustain higher rabbit populations each month on average. Area B would probably be more susceptible to this effect than Area A because of its lower growth rate. If this occurred, it would also suggest that the removal methods changed how carrying capacity affected population growth in each area.**

If poison killed fewer rabbits than it should, it might not have the same effect. Say if poison killed 5% fewer rabbits. **Poison would probably kill fewer rabbits in Area A. Poison's effect varied widely on Area B in the experiments. Like hunting, poison might even stimulate population growth and cause Area B's population to jump past its current number of rabbit months.**

Even if disease or hunting killed 5% less rabbits, probably neither would kill less rabbits than poison. However, hunting might kill slightly fewer rabbits in Area A and Area B because more rabbits would be alive to compete for resources in general. Overcrowding would then have more of an effect on each area.

Predicted Effects of Less Rainfall

Table 2 shows the results for experiments that assume lower rainfall in each area. Each experiment uses an estimated *R* that assumes lower rainfall. Like the first set of experiments, these experiments include the transient phase. Based on these experiments, less rainfall would cause these changes in the previous results:

- **Poison would remove 3726 rabbits from Area A and increase Area B's rabbit population by 248. Without a change in rainfall, poison would remove 2039 less rabbits from Area A. As previously stated, poison would normally decrease the rabbit population of Area B by 1260.**
- **Hunting would increase the number of rabbits in Area A by 8363 and by 8838 in Area B. Without a change in rainfall, hunting would add 244 less rabbits to Area A and 2772 more rabbits to Area B.**
- **Disease would remove 4593 rabbits from Area A and 3958 rabbits from Area B. Without a change in rainfall, disease would remove 3697 fewer rabbits from Area A. As previously stated, disease would normally add 58 rabbits to Area B.**

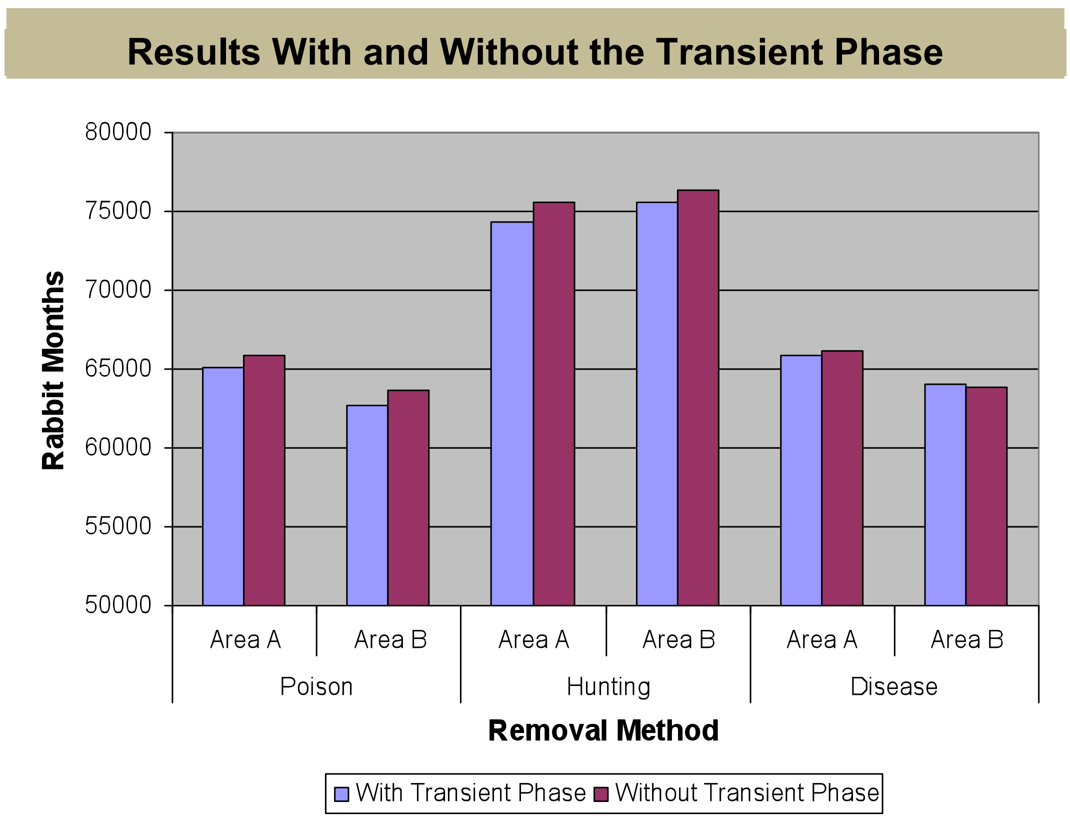

Fig. 3: *The experiments in Table 1 include the first ten months, the transient phase. This chart compares the estimates from Table 1 to the estimates from the second set of experiments, which ignore the transient phase.*

Rabbit Months in a Drier Climate

Removal Method	Area A	Area B
Poison	63076	64208
Hunting	75165	72798
Disease	62209	60002

Table 2: *Table 1 shows the results for the experiments measuring the effects of each removal method under drier conditions. These results include the first ten months, the transient phase. Results for each experiment appear in rabbit months, the sum of the monthly rabbit populations over the span of the experiment.*

Based on the results from the two parts, a human-introduced disease would have the best chance of removing rabbits from Area A and Area B. The experiments taking into account normal rainfall imply that disease could remove rabbits from both areas under wetter conditions. Under drier conditions, disease would remove more rabbits in both areas than poison. Experiments using the minimum R and maximum R based on reduced rainfall also imply that disease would remove the most rabbits from each area under drier conditions. (For the minimum R, disease would 62209 rabbits. For the maximum R, disease would remove 62822 rabbits.)

If the rainfall drops below 40 cm/year, the climatologists' analysis will be worth 7654 rabbits (based on the number of rabbits poison would remove under wetter conditions). If the rainfall does decrease, but the government still uses a disease, Area A and Area B together would have a total of 2109 more rabbit months.

The government should hire more disease biologists. While Area A's chaotic population dynamics cast uncertainty on all of the predictions, a recommendation needs to take into account that annual rainfall has a 75% of decreasing by 10 cm over the next decade. Area B serves as the deciding factor because predictions about its rabbit population will be more reliable. At worst, the climate will stay wetter, and disease will have a limited effect on the Area B's rabbit population. At best, the conditions will get drier, and disease will remove a significant number of rabbits from Area B.

Before climatologists stated the probability for a change in annual rainfall over the next decade, a change in rainfall had a 50% chance of occurring. This amount of information can be delivered in one bit. Their probability for a decrease in rainfall takes .8113 bits to express. **Thus they have given .1887 bits of information about the amount of rainfall.**

Summary

Based just on the experiments that test the removal methods under wetter conditions, poison would remove the most rabbits in both areas. Based on the experiments that test removal methods under drier conditions, disease would remove the most rabbits in both areas. Disease still would probably remove rabbits from both areas even under wetter conditions, so the government should invest in disease biologists.

References

[1] M. Mitchell. "Dynamics, Chaos, and Prediction" in *Complexity: A Guided Tour*. New York: Oxford University Press, 2009, pp. 15-39.