Project 1: Logistic Map & Chaotic Dynamics
Complex Adaptive Systems
CS 423/523 Spring 2013
Professor Melanie Moses
UNM Computer Science

The Rules:
Turn in a hardcopy of your report at the **START of class on February 4.**

Turn in electronic copies of code and report to Matthew Fricke by the start of class on Feb 4. He will provide specific instructions.

You will work in pairs. Anyone without a partner should stay after class on Wed 1/23 to find one. I will help you if needed.

You may talk to other students and discuss the questions, but you may NOT share code that you write to generate results or figures, and you must come up with your own description of your work.

You may alter code you find online (EXCEPT NOT for Problem 4 which must be written from scratch); however, you must DOCUMENT the source of all code that you incorporate into your analysis or report. If you extend, modify, or completely re-write code, anything from changing variable names to modifying the control structure of the code—no matter what you have done to the code, you MUST DOCUMENT the source of all code you did not write yourself. You must also explain what you did to modify it in the Methods section.

You may program in whatever language you like, but you must provide a README file providing exact steps to run your code. If libraries are required for your code to run, the README must make it obvious and easy for the grader to get those libraries.

I will not comment on draft versions of your reports by email. However, you are encouraged to come to office hours with questions, and you may bring a draft of your report.

Note that the programming for this assignment is meant to be very simple (easier than it will be for the next 2 assignments). The difficult part will be presenting your results clearly. Most of your time should be spent on the presentation of your results: make your sentences clear and without grammatical errors, make your figures legible, and summarize relevant data in meaningful tables. Effective communication in a clear and concise report is just as important as valid answers.

The Assignment

Part 1: Analyze the logistic map

1. Create a Figure 1 which shows predicted **NUMBER** of rabbits in a population vs. time, given R = 3.25, K = 200 (K is carrying capacity), and each time step for the logistic map set to 1 month (you can assume that the rabbit generation time is 1 month). Plot 50 time steps. Show on Figure 1 results for two separate rabbit populations, Population A in which x_{0a} = 0.3, and Population B for which x_{0b} = 0.7 (R and K are the same for both populations.)

   Discuss whether the two populations become synchronized, and how long it takes if it occurs.

   List 3 assumptions that your predictions rely on.

2. Create Figure 2 exactly like Figure 1, but with R = 3.7.
   Discuss whether the two populations become synchronized, and how long it takes if it occurs.

   Explain conditions under which predicted population dynamics can become synchronized even with different initial conditions.

   Do you expect to see synchronized dynamics in real populations?

   Explain why the population dynamics in Figure 2 are more difficult to predict than Figure 1.

3. Create another figure, Figure 3, that demonstrates what sensitive dependence on initial conditions is, and the circumstances under which the precision of numerical representation affects predictions. Figure 3 can have multiple panels. Provide an explanation in your text that refers to Figure 3.

4. Write code to create a bifurcation diagram in Figure 4a. Label your axes and define all variables in your axes in your caption. Do not just download a picture of a bifurcation diagram. You must write YOUR OWN code that generates the figure. Your methods must contain a paragraph describing how you generated the bifurcation diagram.

   Create two additional figures, Figure 4b and Figure 4c that "zoom in" to show the detail of the bifurcation diagram demonstrating its fractal nature (you can focus on period doubling regions (i.e. from 1 to 2, 2 to 4, or 4 to 8). Also create a Figure 4d that shows stability within the chaotic region.
   Be sure to properly label your axes. Refer to your figures in your text to explain
   a) the transition to chaos in the logistic map
   b) Feigenbaum's constant
   c) regions of order within chaos
   d) fractal structure in the logistic map

5. We can define how much uncertainty or randomness there is in a dataset using Shannon Entropy. Explain how the entropy in the number of individuals in a population changes as R increases from 3 to 4. You can restrict your answer to asymptotic population values (i.e. after the population has converged to an equilibrium value, or a set of periodic values, if it ever does.) You may refer to any of your previous figures in your explanation.

Part 2: Analyze time series

6. Assume Time Series X and Time Series Y (posted online) were generated by a logistic map. Estimate x_0, R and K for each.

Hints: Use your bifurcation diagram to relate the values of x_t in each time series to possible values of R. For both time series, K (carrying capacity) is a multiple of 100. x_0 and R can be estimated to a single decimal place (e.g. R might equal 3.80, but it does not equal 3.823794).

Be sure to explain how you estimated x_0, R and K in the Methods section of your report.

Can you reproduce these time series exactly? If not, why not. Predict their values at 5 time steps later (at 105 days) and 20 time steps later (at 120 days). How confident are you in your predictions?

7. Time series Z is a real dataset, thought to estimate the population dynamics of Canadian Lynx (which, coincidentally, like to eat rabbits.) Describe any periodicity you see Time Series Z. You may do this by defining population peaks as times when the population exceeds a threshold, and the subsequent population is lower. (Extra credit if you use a Fourier transform to find periods in the data set.)

Provide 4 reasons why Time Series Z can not have been produced by the logistic map. (Hints—consider the heights of the peaks, the differences between peak and lowest population values, the value of K, whether the dynamics are chaotic or periodic.) You may refer to earlier figures or provide new figures to explain your reasons (but new figures are not necessary).

Report Format

Your reports should be approximately 5 pages (no more than 10), double spaced, 11 or 12 point font.

Figures should be approximately $1/6^{th}$ of a page and integrated into the text. Every figure must include a title, meaningful axis labels, tic marks, legible font sizes, a title and a descriptive caption.

The report must contain these sections:

Author Contributions: 1 short paragraph explaining what each team member contributed to the code, analysis and writing. Note that each team member should contribute some to each of these. For example "Moses wrote the code for problems 1 - 3, Fricke wrote the code for problems 4 and 6, and both worked together to answer Problem 5. Moses wrote the first draft of the report. Fricke proof-read and revised the report and figures to make them more clear."

If a student outside of your group helped you in a substantial way, explain that here.

Introduction: A 2 – 3 paragraph description of the goals of your report, definitions of the logistic map and chaos, and why they are relevant to Complex Adaptive Systems.

Methods: Methods should describe your code and analysis of data in sufficient detail that someone could read your report and reproduce your results. Your methods will largely refer to your attached code for details, for example, "We call the logmap function in file XX to produce the time series in questions 1 and 2." Be sure to provide several sentences explaining how you created Figure 4, and how you came up with your estimates for Problem 6.

Results: Your figures and explanations go in this section. Your grade will depend on whether your explanations and figures are correct, clear and concise.

Conclusions: 1-2 paragraphs that summarize your results, explain when the logistic map is useful for predicting population time series, and list any caveats that the reader should consider about the logistic map.

References: All journals or books that you refer to should be cited in IEEE format, e.g.:

[1] W.K. Chen, *Linear Networks and Systems*. Belmont, CA: Wadsworth, 1993, pp. 123-35.
[2] G. Pevere, "Infrared Nation," *The International Journal of Infrared Design*, vol. 33, pp. 56-99, Jan. 1979.

Additional examples available at: http://library.queensu.ca/book/export/html/5846

Refer to your citations with a bracketed number in the main text, for example "We chose values for R based on the fractal region shown Figure X.X in [2]."

You should document code that you acquired and modified by listing an author, web page, and the date that you accessed it.