

Logistic Map



n rabbits on an island, that holds k rabbits
Rabbits are born at rate b and die at rate d

$$1) n_{t+1} = (b-d)(n_t - n_t^2/k)$$

As n approaches k , the growth rate slows

$$2) \text{ Rewrite (1) as } n_{t+1} = (b-d)(kn_t - n_t^2)/k$$

$$3) \text{ let } x_t = n_t/k \text{ and } R = b-d$$

$$\text{LOGISTIC MAP: } x_{t+1} = Rx_t(1-x_t)$$

(SEE MATLAB)

(in Flake, CoBN $x_{t+1} = 4rx_t(1-x_t)$, so $R = 4r$)

k is carrying capacity

n_t = num rabbits at time t

x_t is fraction of carrying capacity
at time t

R is intrinsic growth rate: $b-d$

Chaos

- Deterministic: No randomness in the equations, predictable given perfect information
- Sensitive: to initial conditions & perturbations
- Ergodic: the system will eventually return to someplace near where it's been in the past
- Embedded: there are regions of stability and periodicity within the chaos
- Surprising!
 - Random behavior can emerge from deterministic systems with no external randomness
 - Behavior of simple systems is impossible to predict *in principle* as well as practice
 - There is order in the chaos: Feigenbaum's constant

Feigenbaum's constant

$R_1 \sim 3.0$ ($2^1 = 2$ period attractor)

$R_2 \sim 3.44949$ ($2^2 = 4$ period attractor)

$R_3 \sim 3.54409$ ($2^3 = 8$ period attractor)

$R_4 \sim 3.564407$

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$R_{\text{inf}} \sim 3.569946$

The rate at which the R values converge is Feigenbaum's constant ~ 4.6692016

Feigenbaum's constant is the same for all 'unimodal maps'

Period doubling route to chaos

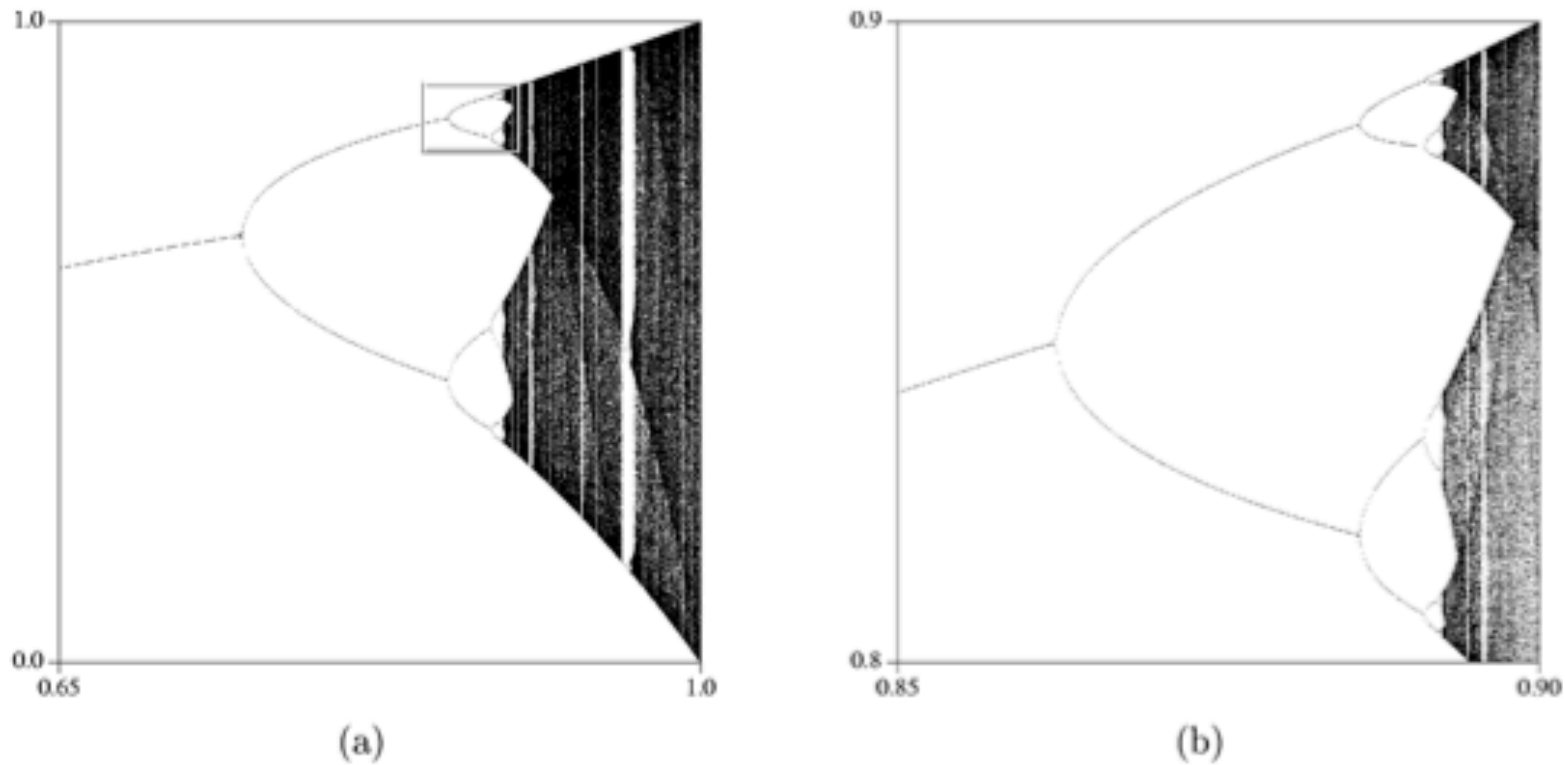


Figure 10.7 Bifurcation diagrams for the logistic map: (a) This image has values of r such that fixed points, limit cycles, and chaos are all visible. (b) This image shows the detail of the boxed section of (a).

Figure from *The Computational Beauty of Nature: Computer Explorations of Fractals, Chaos, Complex Systems, and Adaptation*. Copyright © 1998–2000 by Gary William Flake. All rights reserved. Permission granted for educational, scholarly, and personal use provided that this notice remains intact and unaltered. No part of this work may be reproduced for commercial purposes without prior written permission from the MIT Press.

- Flake:
 - Stability of fixed points
 - Pastry: stretch and fold