Logistic Map

n rabbits on an island, that holds k rabbits
Rabbits are born at rate b and die at rate d

1) $n_{t+1} = (b\text{-}d)(n_t - n_t^2/k)$
As n approaches k, the growth rate slows

2) Rewrite (1) as $n_{t+1} = (b\text{-}d)(kn_t - n_t^2)/k$

3) let $x_t = n_t/k$ and $R = b\text{-}d$
LOGISTIC MAP: $x_{t+1} = Rx_t(1\text{-}x_t)$
(SEE MATLAB)

(in Flake, CoBN $x_{t+1} = 4rx_t(1\text{-}x_t)$, so $R = 4r$)

k is carrying capacity
n_t= num rabbits at time t

x_t is fraction of carrying capacity at time t

R is intrinsic growth rate: b-d

Chaos

- Deterministic: No randomness in the equations, predictable given perfect information
- Sensitive: to initial conditions & perturbations
- Ergodic: the system will eventually return to someplace near where it's been in the past
- Embedded: there are regions of stability and periodicity within the chaos
- Surprising!
  - Random behavior can emerge from deterministic systems with no external randomness
  - Behavior of simple systems is impossible to predict *in principle* as well as practice
  - There is order in the chaos: Feigenbaum's constant

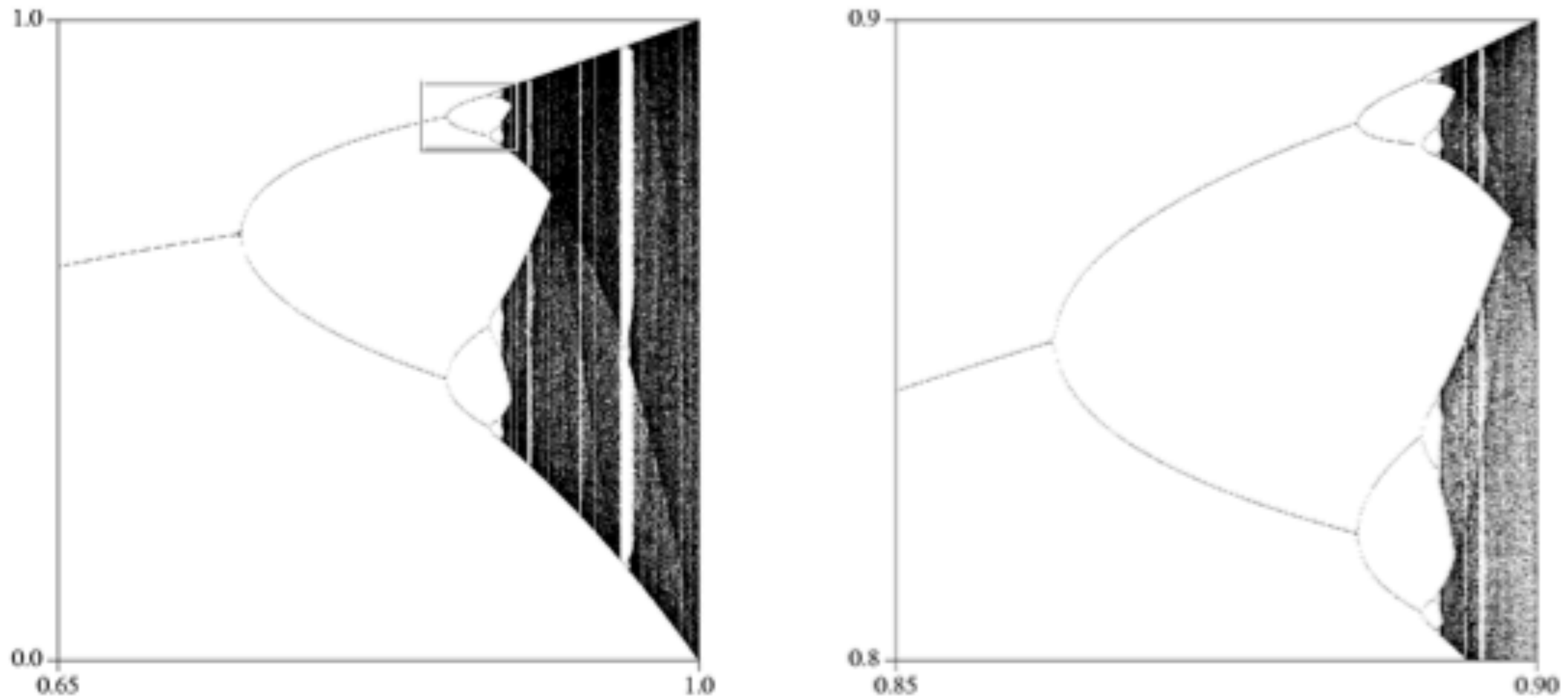
Feigenbaum's constant

R_1 ~ 3.0 (2^1 = 2 period attractor)
R_2 ~ 3.44949 (2^2 = 4 period attractor)
R_3 ~ 3.54409 (2^3 = 8 period attractor)
R_4 ~ 3.564407
.
.
.
R_{inf} ~ 3.569946

The rate at which the R values converge is Feigenbaum's constant ~ 4.6692016

Feigenbaum's constant is the same for all 'unimodal maps'

Period doubling route to chaos



(a) (b)

Figure 10.7 Bifurcation diagrams for the logistic map: (a) This image has values of r such that fixed points, limit cycles, and chaos are all visible. (b) This image shows the detail of the boxed section of (a).

- Flake:
  - Stability of fixed points
  - Pastry: stretch and fold