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Abstract

Endowing artificial swarms with collective decision-making capabilities has been a major endeavor in swarm intelligence research. Researchers working in this area have drawn inspiration from the behavior of social insects. In this paper, we introduce a collective decision-making mechanism for swarms that is not inspired by any social insect behavior. Our proposed mechanism is based on an opinion formation model. Opinion formation models, studied mostly in statistical physics, try to capture the essential mechanisms of agreement in large populations of interacting agents. We use the majority-rule opinion formation model in which agents are in one of two states called opinions. Agents change opinion through the repeated application of the majority rule on small teams of agents. We extend this model with the concept of *differential latency*. With this extension, agents that just adopted an opinion go into a latent state during which they cannot influence nor be influenced by other agents. The duration of a latent state is stochastic and depends on the opinion adopted by an agent. Consensus on the opinion associated with the shortest average latency is the result of this extension. In a swarm intelligence context, this means that agents whose opinions represent actions that take time to perform, will choose by consensus the action that takes less time to perform. We validate our proposal with a swarm robotics experiment in which robots must choose one of two paths of different length that connect two locations. As a result of the application of the proposed mechanism, the swarm is able to choose the shortest path.

1 Introduction

Natural swarms are known for their ability to make good collective decisions in a completely decentralized way. A collective decision, sometimes also called a consensus decision, occurs when all the members of a group choose the same action from a set of two or more mutually exclusive alternative actions (Conradt and Roper, 2005). Well-known examples of collective decisions are the ones made by swarms of bees when selecting the best nest site from a number of candidates (Seeley, 2010), or by ant colonies when selecting the shortest path from their nest to a food source (Goss et al., 1989). Researchers working in the swarm intelligence field (Bonabeau et al., 1999; Dorigo and Birattari, 2007), try to endow artificial swarms with collective decision-making capabilities similar to the ones observed in natural swarms. The main reason why decentralized collective decision-making is a desirable feature in artificial swarms is that it can be as advantageous as it is in natural swarms (Kordon, 2010; Krause and Ruxton, 2002).

In this paper, we introduce a mechanism that allows swarms to exhibit collective decision-making capabilities.[1] The novelty of our proposal is that we do not take inspiration from the behavior of social insects as it is common in the field (Bonabeau et al., 2000; Şahin, 2005). Instead, we look at the phenomenon of collective decision-making in swarms as a process of agreement in a large population of interacting physical agents. This viewpoint leads us to consider opinion formation models, which are usually discussed in the statistical physics literature (see Castellano et al. (2009), for a recent survey), as possible agreement mechanisms. Opinion formation models are used for modeling large-scale social, economic, and natural phenomena that involve many interacting agents.

The collective decision-making mechanism introduced in this paper is based on two components: *a)* an opinion formation model that evolves through the repeated use of the majority rule, and *b)* properties of swarm intelligence systems composed of embodied agents. The majority-rule opinion formation model, proposed by Krapivsky and Redner (2003), operates on a population of agents each of which is in one of two possible states, called opinions. The dynamics of the model result from the iterative application of the majority rule on teams of randomly picked agents from a large population. When the population of agents is well-mixed, that is, when an agent can be teamed up with any other agent in the population, the majority-rule opinion dynamics make the population reach a consensus, that is, a state in which all agents share the same opinion. Some of the properties of swarm intelligence systems that together with the majority-rule opinion formation model produce collective-decision making capabilities are: *i)* different agents can act in parallel, and *ii)* agents' actions take time to perform. These properties make us extend Krapivsky and Redner's model in a number of ways. The most significant extension that we introduce is the concept of *differential latency*. This means that when an agent adopts an opinion, it becomes *latent*, that is, it cannot change opinion again for a period of time of stochastic duration. The mean duration of this *latent state* depends on the agent's recently adopted opinion.

The potential of the proposed collective decision-making mechanism is demonstrated through a swarm robotics task in which teams of robots must transport objects from a source location to a target location. These two locations are connected through two paths of different lengths. Teams need to choose between these two paths to transport the objects. Different from previous approaches, which require robots to measure path-travel times or to rely on pheromone-like information, we simply associate with the available paths the two opinions the majority-rule opinion formation model is able to work with. The result is an intelligent collective decision without intelligent decision-makers. The swarm of

[1] A preliminary version of this proposal has been published in (Montes de Oca et al., 2010).

robots eventually selects the shortest path.

The contributions of our work are:

1. An extension of the majority-rule opinion formation model proposed by Krapivsky and Redner (2003). We introduce: *a)* multiple teams of agents, and *b)* different latency period lengths depending on the adopted opinions, that is, we introduce the concept of differential latency.

2. An analysis of the system's dynamics when latency periods are exponentially distributed. This analysis shows that the population of agents reaches consensus on the opinion associated with the shortest latency period.

3. A study in simulation of the system's dynamics when latency periods are normally distributed. In this case, we determine the conditions under which the population of agents reaches consensus on the opinion associated with the shortest latency period.

4. An application of the proposed extended model on a swarm robotics scenario. We show that the dynamics of the majority-rule opinion formation model with differential latency can be used as a decentralized collective decision-making mechanism.

The rest of this paper is structured as follows. In Section 2, we describe the opinion formation models that are the direct precursors of our proposal. In Section 3, we present the extensions introduced in this paper, the analysis of the model's dynamics when latency periods are exponentially distributed, as well as the study in simulation when latency periods are normally distributed. In Section 4, we present the application of the extended model to a swarm robotics scenario as an example of the potential of the approach as a decentralized collective decision-making mechanism. In Section 5, we discuss the differences and similarities between our approach and previous works in the specialized literature. We also discuss limitations of the proposal as well as ways to overcome them. We conclude our presentation in Section 6.

2 Opinion Formation Based on the Majority Rule

In this section, we describe two opinion formation models based on the majority rule. These two models are the direct precursors of our proposal, which is described in Section 3.

2.1 Majority-Rule Opinion Dynamics without Latency

Krapivsky and Redner (2003) studied the opinion dynamics that results from the iterative application of the majority rule in a population of N agents each of which can be in one of two possible states, called opinions (A or B)[2]. The number of agents with opinion A is denoted by a. Likewise, the number of agents with opinion B is denoted by b. It is always true that $N = a + b$. Agents can change opinion over time depending on the opinion held by the majority of the team they belong to. The model's initial condition is the density of agents

[2]Throughout this paper, we use letters A and B to label the two available opinions.

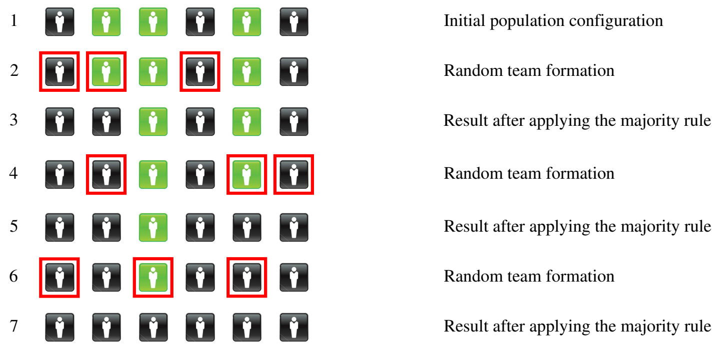
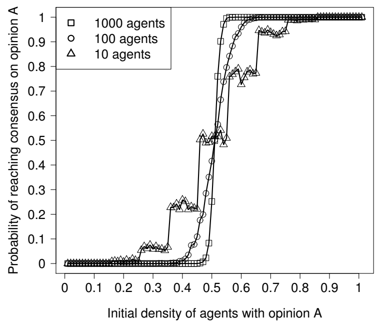


Figure 1: Example of the dynamics induced by the iterative application of the majority rule on a population of six agents. Agents' opinions are represented by different shades or colors. Initially, the density of agents in favor of the opinion represented by black is 0.5. After applying three times the majority rule on randomly-formed teams of three agents each (marked with squares), the population reaches consensus with on one of the two competing opinions.

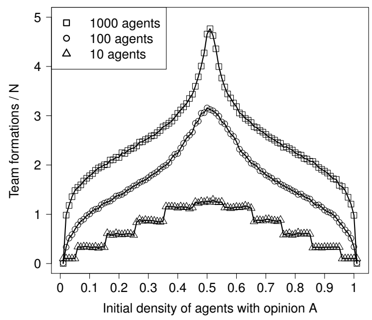
with opinion A at time step $t = 0$. This density is defined as $\rho_A = a/N$. The evolution of the system proceeds as follows: three different agents are randomly picked from the population to form a team. The majority rule is then used to aggregate the opinions of the team members and transform them into a single opinion that all team members adopt. Team members are put back in the population, and a new team made of randomly picked agents is formed. The process is repeated until consensus is reached on one of the two available opinions. Figure 1 shows a depiction of the process just described.

Krapivsky and Redner (2003) showed that when the population is well-mixed, that is, when any three agents in the population can form a team, the population eventually reaches a consensus. They also showed that the opinion on which the population reaches consensus depends on the initial conditions. An example of this is shown in Figure 2(a). If the initial absolute majority (i.e., the majority at the level of the population) favors one opinion, say A, over the other, the population reaches consensus on opinion A with a higher probability than on opinion B. If there is no initial absolute majority, the opinion on which consensus is reached is either A or B. The relation between the initial density of individuals favoring one opinion over the other, and the probability to reach consensus on one of those opinions is nonlinear. Small deviations of the initial opinion density from the critical value of 0.5, produce large changes in the probability of reaching consensus on one or the other opinion. The size of the population is another factor that affects the dynamics of the system. The larger the population size, the sharper is the transition of the probability of reaching consensus on the initial absolute majority. In other words, the system is more sensitive to the initial absolute majority when the opinion dynamics occur on a large population.

The number of team formations needed to reach consensus as a function of the initial opinion density and the population size is shown in Figure 2(b). The time required to reach consensus in the majority-rule opinion formation model



(a) Probability of reaching consensus on one opinion



(b) Number of team formations to reach consensus

Figure 2: Characteristic dynamics of the majority-rule opinion formation model. The probability of reaching consensus on one opinion (labeled A) as a function of the initial density of agents in favor of that opinion and the population size is shown in (a). The number of team formations needed to reach consensus as a function of the initial opinion density and the population size is shown in (b). These plots are based on data gathered through 1,000 independent runs of a Monte Carlo simulation.

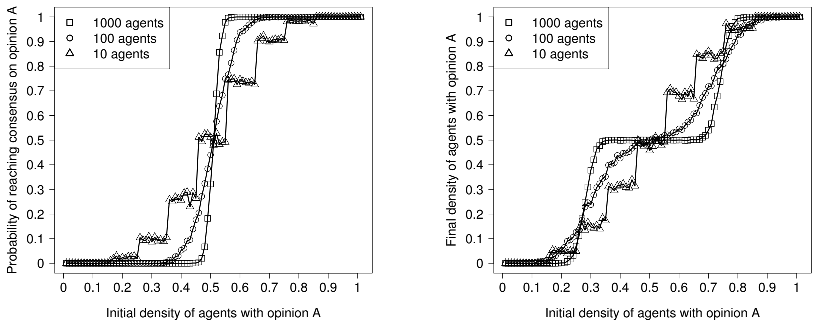
grows nonlinearly with the population size. There is also a critical value of the initial opinion density. When this quantity is equal to 0.5, a maximum in the number of team formations needed to achieve consensus is reached.

2.2 Majority-Rule Opinion Dynamics with Latency

Lambiotte et al. (2009) introduced *latency* into Krapivsky and Redner's model. Latency is a period of time of stochastic duration during which an agent cannot be influenced by other agents, and thus cannot change opinion. The evolution of the system proceeds as follows: three different agents are randomly picked from the population to form a team. The opinion shared by the majority of the team members is identified, but team members decide whether to adopt the team's majority opinion depending on whether they are in a latent state or not. If an agent is not in a latent state, it adopts the team's opinion and if this opinion is different from its previous opinion, the agent goes into a latent state. If an agent was already in a latent state when the team was formed, the agent ceases to be in a latent state with probability λ, which is a parameter of the model.

In Lambiotte et al.'s model, the resulting dynamics do not always lead to consensus. Depending on the probability of an agent becoming non-latent, λ, a so-called "zero magnetization" state is also possible. In a zero magnetization state the proportion of agents favoring one opinion or another fluctuates randomly and no opinion is favored in the long run. The characteristic dynamics of Lambiotte et al.'s model are shown in Figure 3.

In Lambiotte et al.'s model, the tendency to reach consensus induced by the majority rule is cancelled by the possibility of latent agents to influence non-latent agents without being influenced themselves. This combination of effects can inhibit consensus formation and lead to a zero magnetization state (Lam-



(a) Resulting dynamics when $\lambda = 1/2$ (b) Resulting dynamics when $\lambda = 1/20$

Figure 3: Characteristic dynamics of Lambiotte et al.'s extension of the majority-rule opinion formation model. Depending in the value of the probability λ consensus may or may not be achieved. For example, when $\lambda = 1/2$, consensus is always achieved. Thus, in Figure (a), we plot the probability of reaching consensus on opinion A as a function of the initial density of agents in favor of that opinion and the population size. In contrast, when $\lambda = 1/20$, the population do not always achieve consensus. In Figure (b), we plot the average density of agents with opinion A after 100,000 team formations. These plots are based on data gathered through 1,000 independent runs of a Monte Carlo simulation.

biotte et al., 2009).

3 Majority-Rule Opinion Dynamics with Differential Latency

In this section, we present the extensions introduced in this paper to the majority-rule opinion formation models proposed by Krapivsky and Redner (2003) and Lambiotte et al. (2009). We begin by describing the features of real-world multi-robot systems that motivate these extensions. Then, we analyze the dynamics of the system when latency periods are exponentially distributed. We finish this section with a study of the system's dynamics when latency periods are normally distributed. The first analysis is of theoretical interest as it is a direct successor of Lambiotte et al.'s model. The second analysis is an attempt to study the dynamics of the system under more realistic conditions.

3.1 The Proposed Model

Before we describe the extensions to Krapivsky and Redner's and Lambiotte et al.'s models that we propose in this paper, it is important to define first the fundamental connection between those models and a swarm robotics system: opinions can represent actions or sets of actions that robots have to choose from while executing a task. Examples of single actions that can be modeled as opinions are whether to turn left or right at some point in an environment, or whether to connect or not with another robot. Examples of an opinion representing more than one action could be whether to follow the rules for moving with other robots in a formation, or whether to follow the rules for assembling one shape or another for a specific task.

The extensions introduced to Krapivsky and Redner's and Lambiotte et al.'s models are motivated by some properties of real-world swarm robotics systems. These properties and the corresponding extensions are the following:

1. A swarm robotics system is composed by many robots which can operate in parallel. This property is directly translated into k independent teams instead of just one as in the original formulations of Krapivsky and Redner's and Lambiotte et al.'s models.

2. Robot actions may require physical displacement. Since in swarms of robots communication is local, physical displacement means that robots that are executing an action cannot be influenced by other robots, and crucially, cannot influence other robots. The fact that robots cannot be influenced by other robots, and therefore, cannot change opinion after forming a team, is already captured by the concept of latency as defined by Lambiotte et al.. However, in Lambiotte et al.'s model latent agents can still influence other agents. Thus, we extend Lambiotte et al.'s model by restricting team members to be non-latent at the moment of forming a team. This change forbids latent agents to influence other agents.

3. Robot actions take time to perform. Moreover, the duration of an action is stochastic because there are physical interactions between robots and the environment. In addition, different actions may have different average duration. This is translated into *differential latency*, that is, the average duration of the latency period depends on the agents' recently adopted opinion. In contrast with Lambiotte et al.'s model in which agents go into a latent state only if they switch opinions, in our case, agents go into a latent state regardless of whether they switched opinion or not. They always go into a latent state after the team they belong to makes a decision.

The way the system evolves is similar to the way it does in Krapivsky and Redner's and Lambiotte et al.'s models. First, teams are formed at random. Then, each team applies the majority rule to update its members' opinions. After that, all agents that belong to a team enter into a latent state whose duration depends on the team's recently adopted opinion. Finally, once a team leaves the latent state, its members become available again to form a new team. The new team members will be picked from a reduced population because, at that point in time, there will be latent agents that belong to other teams, and thus cannot be picked to form new teams. The process is repeated *ad infinitum* or until the population reaches a consensus. Table 1 shows a pseudo-code version of the process through which the system ideally evolves.

In the remainder of this section, we analyze the opinion dynamics that result from our extensions. We focus on two cases: *a)* when the duration of the latency periods are exponentially distributed, and *b)* when the duration of the latency periods are normally distributed. The fist case is a direct extension of Lambiotte et al.'s model in which the duration of the latency period is exponentially distributed with respect to the number of times an agent becomes part of

Table 1: Majority-Rule Opinion Formation with Differential Latency

Input: Number of agents, N, number of teams, k, initial density of agents with opinion A, $\rho_A = a/N$.

```
/* Initialization */
t ← 0
Initialize population of agents X with density ρ_A.
/* Initial team formations */
for i = 1 to k do
  Form team i by selecting at random three non-latent agents from X.
  Apply the majority rule in team i, updating team members' opinions.
  Team i enters into a latent state according to adopted opinion.
end for
repeat
  for i = 1 to k do
    if Team i leaves latent state then
      Form new team i by selecting at random three non-latent agents from
      X.
      Apply the majority rule in team i, updating team members' opinions.
      Team i enters into a latent state according to adopted opinion.
    end if
  end for
  t ← t + 1
until Ad infinitum or consensus is reached
```

a team. In the second case, we try to model a more realistic situation in which agents are robots that have to perform actions that cannot be completed at a constant rate per unit of time.

3.2 Exponentially Distributed Latency Periods

In this subsection we assume that the delays the agents experience when they execute actions are distributed exponentially. This assumption allows to study the effect of the proposed extensions on the dynamics of the system by means of a differential equation model. More precisely, we assume an unlimited number of agents and model the fractions of agents that favor the different opinions. As shown in Toral and Tessone (2007) in models of opinion dynamics the finiteness of the number of agents may play a crucial role. Nevertheless, the characterization of the models at a macroscopic scale, that is, in the continuum limit, can lead to useful insights (Castellano et al., 2009). It can help to characterize quantitative behaviors, to understand the robustness of certain features and might filter out non-universal microscopic details.

One of the proposed extensions to the majority-rule model is that a fixed number of teams operate in parallel. In the differential equation model this is reflected by a fix proportion between latent and non-latent agents (recall that agents are latent if they are currently executing an action, and non-latent otherwise). Let $0 < \beta < 1$ denote the fixed fraction of all agents that stays non-latent. The fraction of non-latent agents that have opinion A is denoted by $f(t)$ (in the following non-Greek letters denote functions; moreover we omit the time index t, thus we write f instead of $f(t)$). The fraction of agents that are latent and have opinion A is denoted by a. Thus, $f(t) + a(a)$ gives overall the fraction of agents that favor opinion A at time t.

Without loss of generality the mean time agents with opinion A stay latent is set to 1 and the mean time agents with opinion B stay latent is set to $1/\lambda$ with $0 < \lambda \leq 1$. Thus $\lambda = 1/2$ implies, for example, that agents with opinion B stay in mean twice as long latent than agents with opinion A (action A takes twice as long that B).

Within a unit time step a certain fraction of agents leaves the latent state. Because the time the agents stay latent is distributed exponentially, this fraction is proportional to a for agents with opinion A and to $\lambda(1 - \beta - a)$ for agents with opinion B. Hence, the overall number of agents that become non-latent is

$$r = a + \lambda(1 - \beta - a).$$

If agents finish their actions immediately new teams are formed. Consequently, if in the continuum model r agents leave the latent state the same fraction of agents enter it. These r agents are chosen randomly from the β non-latent agents. The probability to choose an agent with opinion A among these agents is $p = f/\beta$. Hence, the fraction of agents that are non-latent and favor opinion A decreases by rp in a unit time step. A team of three randomly chosen non-latent agents enter the latent state with opinion A if at least two agents favor A. Thus, the probability that a team enters latent state with opinion A is given by $\binom{3}{2}p^2(1-p) + p^3$. Adding these parts together, the dynamics of the system can be modeled as:

$$\begin{aligned} \dot{f} &= -rp + a \\ \dot{a} &= -a + r(3p^2 - 2p^3) \end{aligned} \tag{1}$$

Figure 4(a) depicts six example trajectories of the model. The starting conditions for the trajectories are chosen so that for a given initial fraction of agents with opinion Athe values of f and a are $f = \beta(f+a)$ and $a = (1-\beta)(f+a)$ (the fraction of non-latent agents is $\beta = 0.25$). First consider the case of equal latencies for both opinions ($\lambda = 1$, dashed lines). If the system starts without any bias, that is, with $f + a = 0.5$ (middle line) no evolution takes place. On the other hand, if the system starts biased it finds ultimate consensus on the opinion that was initially in the majority. For example, if the system starts with the majority favoring opinion A ($f + a = 0.52$, top line) it develops consensus on A. If, on the other hand, opinion A is in the minority ($f + a = 0.48$, bottom line) the ultimate consensus is B. This resembles the results from Krapivsky and Redner (2003), where the mean-field limit of the majority rule opinion dynamics model without (differential) latencies is studied.

The introduction of opinion dependent latencies has a strong impact on the dynamics of the system: the system can end up in consensus on an opinion even if this opinion was initially favored only by a minority of the agents. For instance, consider the case that agents with opinion B stay twice as long latent than agents with opinion A ($\lambda = 0.5$, solid lines of Figure 4(a)). In this case it

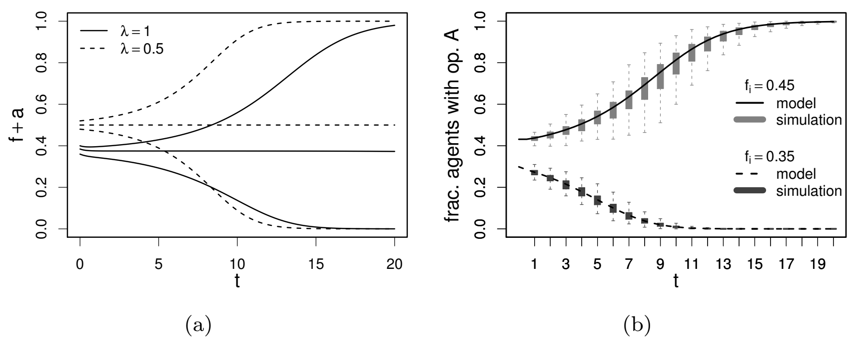


Figure 4: (a) predicted fraction of agents with opinion A over time; for equal latencies (dashed) starting with $f+a = 0.52$ (top),$f+a = 0.5$ (middle),$f+a = 0.48$ (bottom) and differential latencies ($\lambda = 0.5$, solid) starting with $f+a = 0.40$ (top),$f+a = 0.3845$ (middle),$f+a = 0.36$ (bottom); (b) Comparison of the model with an agent based simulation of 1000 agents

is sufficient that an initial fraction of only $f + a = 0.4$ favors opinion A to lead the system to consensus on this opinion (top line).

Figure 4(b) compares the continuum model with results gained in a Monte Carlo simulation. Shown is the fraction of agents that favor opinion A depending on the elapsed time t. The simulation results were obtained with 1000 agents and over 100 independent runs. The depicted bars mark the deviation of the number of agents with opinion A at this time point. For the continuum model the curves give the value of $f(t) + a(t)$, that is, the sum of latent agents and non-latent agents with opinion A. The shown results were obtained with two different initial conditions that were determined as follows. Let f_i be the initial bias (initial fraction of agents with opinion A), then $f(0)$ and $a(0)$ are calculated as $f(0) = \beta f_i$ and $a(0) = (1-\beta)(3f_i^2 - 2f_i^3)$. This calculation is done to take a property of the Monte Carlo simulation into account that is not present in the continuum model. More precisely, in the Monte Carlo simulation all agents start non-latent. In a very first step the desired number of teams is formed and in all of these formed teams the majority rule is applied. Figure 4(b) shows that the predictions of the continuum model fit the simulation results well. For $f_i = 0.35$ the number of agents with opinion A converges to zero. On the other hand, if $f_i = 0.45$ the system ends up with consensus on opinion A.

In the following we will investigate the stability of the equilibrium points of the continuum model. The stationary solutions are the states $[f = \beta, a = 1-\beta]$, $[f = 0, a = 0]$, and $[f = \beta/2, a = \lambda(1-\beta)/(1+\lambda)]$. The two solutions $[f = \beta, a = 1-\beta]$ and $[f = 0, a = 0]$ correspond to consensus on A and B, respectively. The Jacobian matrix evaluated at these points results in $\begin{bmatrix} \frac{\beta-1}{\beta} & \lambda \\ 0 & -\lambda \end{bmatrix}$ and $\begin{bmatrix} \frac{\lambda(1-\beta)}{\beta} & 1 \\ 0 & -1 \end{bmatrix}$, respectively. The eigenvalues of these matrices are $(\beta-1)/\beta$ and $-\lambda$ for the first and $(\lambda\beta - \lambda)/\beta$ and -1 for the second one. All these values have negative real parts (for $0 < \beta, \lambda < 1$) and thus the consensus states are asymptotically stable. On the other hand the analysis shows that the linearization of the system near the third equilibrium point has one positive and

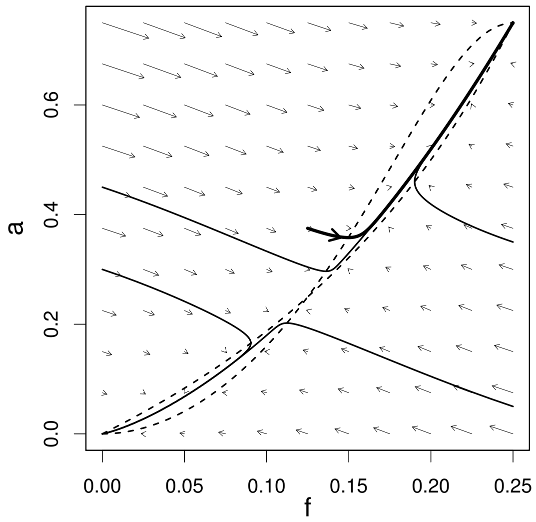


Figure 5: direction field of system (1) with isoclines (dotted) and example trajectories (solid); bold trajectory relates to an unbiased system

one negative eigenvalue (details are omited because of lengthiness). Thus, this point is a saddle point and hence not stable. Consequently, the only stable equilibrium points of the system are $[f = \beta, a = 1 - \beta]$ and $[f = 0, a = 0]$. In other words, in the continuum limit model the agents always find consensus.

Figure 5 visualizes the state space of the system for parameters $\beta = 0.25$ and $\lambda = 0.5$. The dotted lines mark the two isoclines, that is, the solutions of $\dot{f} = 0$ and $\dot{a} = 0$. The saddle point is the point at which the two isoclines meet (the point $[f = 0.125, a = 0.25]$). Moreover, four example trajectories for different starting conditions are depicted. The thick plotted trajectory relates to a "uninformed" system, that is, a system that starts with exactly half of the agents with opinion A.

3.3 Normally Distributed Latency Periods

The analysis presented in the previous section was possible because we assumed that at any given time step agents could switch from a latent to a non-latent state with a constant probability, that is, that the duration of latency periods were exponentially distributed. However, if we reconsider our initial motivation for extending Krapivsky and Redner's and Lambiotte et al.'s models, it is clear that we cannot use exponential distributions to model the time required by a team of robots to finish executing an action. In general, we can expect that action execution times in have a typical duration with some deviation around it. This deviation may or may not be symmetrical with respect to the typical duration.

In this section, we study the dynamics of the system that result from having normally distributed latency periods. Unfortunately, the new conditions do not make the system amenable for an exact analytical treatment. Thus, the analysis that follows is based on Monte Carlo simulation.

3.3.1 Experiment 1: Dynamics

The system is initialized and evolves in exactly the same way as described in Table 1. The only difference is that opinion A is associated with a latency period whose duration is normally distributed with mean μ_A and standard deviation σ_A. Likewise, opinion B is associated with a latency period whose duration is normally distributed with mean μ_B and standard deviation σ_B.

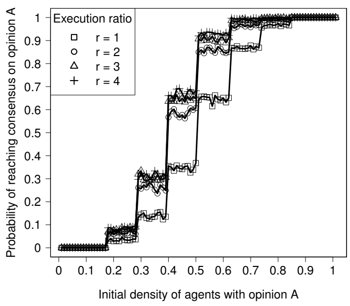
A first set of experiments were run in order to investigate the effects of having different mean latency periods and varying the number of teams. In these experiments, we simulated the dynamics of the system with $N \in \{9, 90, 900\}$ agents and $k \in \{1, 2, 3\}$, when $N = 9$, $k \in \{1, 10, 20, 30\}$, when $N = 90$, and $k \in \{1, 100, 200, 300\}$, when $N = 900$. We varied the latency duration ratio $r = \mu_B/\mu_A$ by changing the value of μ_B. The explored values of r were 1, 2, 3, and 4. The reference mean μ_A was fixed to a value of 100 time steps. We set $\sigma_A = \sigma_B = 20$ time steps. With these settings, the two distributions do not significantly overlap which allows us to see the dynamics in the absence of high levels of interference. Typical results are shown in Figures 6 and 7.[3]

In Figure 6, we show the effects of varying the latency duration ratio, r, while keeping the number of teams, k, constant. In this example, we set $k = N/3 - 1$. The probability of reaching consensus on opinion A as a function of the initial density of agents in favor of that opinion, the population size, and the number of teams is shown in Figures 6(a), 6(c), and 6(e). Opinion A is associated with the shortest mean latency period.

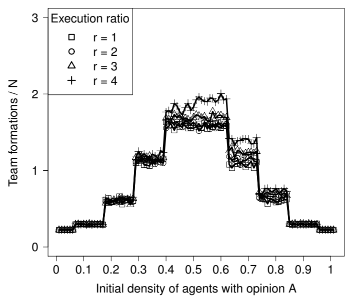
Some aspects of the dynamics of the system with the introduced extensions are similar to those of the original majority-rule opinion formation model. First, the system achieves consensus on one opinion. Second, the actual opinion on which the system achieves consensus depends on the initial density of individuals favoring one opinion over the other. Finally, the relation between the initial configuration of the population and the probability of reaching consensus on one of the alternative opinions follows the same nonlinear pattern. However, there is an important difference between the two systems. When latency periods have a different mean duration, it is more likely that the system achieves consensus on the opinion associated with the shortest mean latency duration than on the other opinion. This is reflected by the shift to the left of the critical initial density in favor of the opinion associated with the shortest mean latency duration. In fact, the larger the latency duration ratio, the smaller is the critical initial density in favor of the opinion associated with the shortest mean latency duration. For example, in Figures 6(c) and 6(e), the critical initial density is approximately equal to 0.35 when $r = 4$, while it is approximately equal to 0.42 when $r = 2$. However, the difference between critical initial densities becomes smaller as the latency duration ratio increases. Thus, it appears there is a lower bound on the value of initial critical density for different latency duration ratios.

The average number of team formations needed to reach consensus under the general conditions of Figure 6 for different population sizes is shown in Figures 6(b), 6(d), and 6(f). In all cases, the maximum number of team formations is reached at the critical initial density. Additionally, at this critical point, the larger the latency duration ratio, the more team formations are needed to reach consensus. Interestingly, the shape of these curves is not symmetric around the

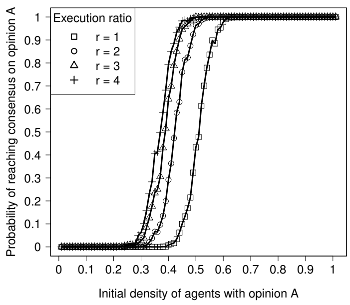
[3] At `http://iridia.ulb.ac.be/supp/IridiaSupp2010-014/` the reader can find the complete set of results and supplemental material for this article.



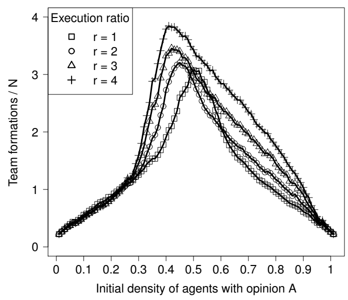
(a) Probability of reaching consensus on opinion A when $N = 9$, and $r \in \{1, 2, 3, 4\}$.



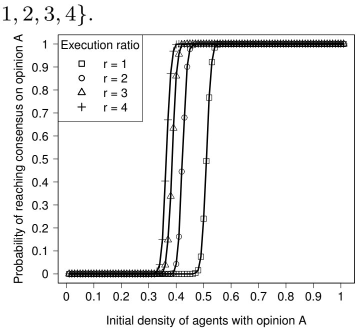
(b) Number of team formations to reach consensus when $N = 9$, and $r \in \{1, 2, 3, 4\}$.



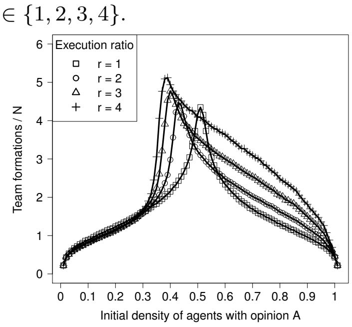
(c) Probability of reaching consensus on opinion A when $N = 90$, and $r \in \{1, 2, 3, 4\}$.



(d) Number of team formations to reach consensus when $N = 90$, and $r \in \{1, 2, 3, 4\}$.



(e) Probability of reaching consensus on opinion A when $N = 900$, and $r \in \{1, 2, 3, 4\}$.

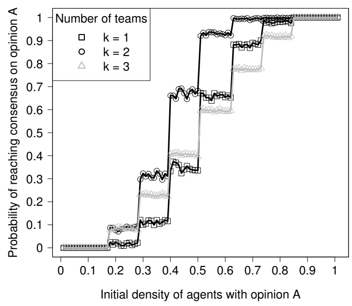


(f) Number of team formations to reach consensus when $N = 900$, and $r \in \{1, 2, 3, 4\}$.

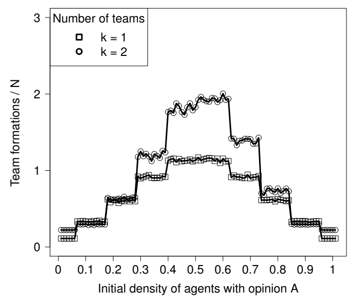
Figure 6: Characteristic dynamics of the majority-rule opinion formation model with normally-distributed latencies. In this figure, we study the effects of varying the latency duration ratio while keeping k constant. In these figures, $k = N/3 - 1$. 1,000 independent runs of a Monte Carlo simulation were used to generate these plots. See text for details.

critical initial density when $r > 1$.

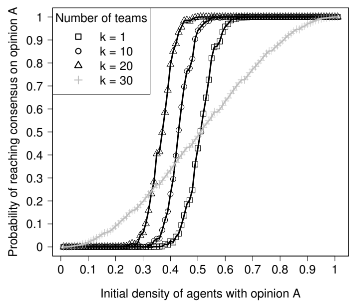
The second aspect that we study in this experiment is the effect on the opinion dynamics when we vary the number of teams. An example of the obtained results is shown in Figure 7. A first result of this experiment is that by varying the number of teams, the likelihood of achieving consensus on the opinion



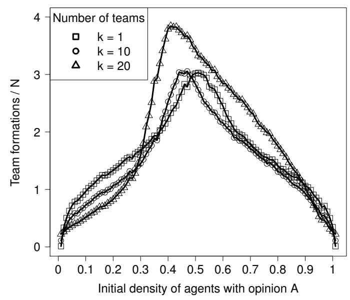
(a) Probability of reaching consensus on opinion A when $N = 9$, and $k \in \{1, 2, 3\}$.



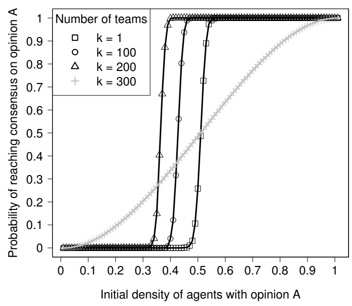
(b) Number of team formations to reach consensus when $N = 9$, and $k \in \{1, 2\}$.



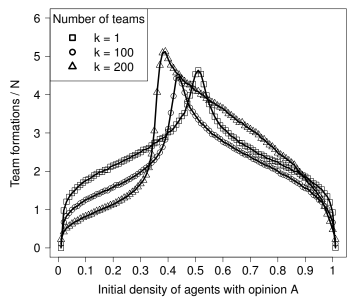
(c) Probability of reaching consensus on opinion A when $N = 90$, and $k \in \{1, 10, 20, 30\}$.



(d) Number of team formations to reach consensus when $N = 90$, and $k \in \{1, 10, 20\}$.



(e) Probability of reaching consensus on opinion A when $N = 900$, and $k \in \{1, 100, 200, 300\}$.



(f) Number of team formations to reach consensus when $N = 900$, and $k \in \{1, 100, 200\}$.

Figure 7: Characteristic dynamics of the majority-rule opinion formation model with normally-distributed latencies. In this figure, we study the effects of changing the number of teams. For these figures we kept $r = 4$. When $k = N/3$, the system does not evolve, and thus, the population does not reach consensus (curve in gray). 1,000 independent runs of a Monte Carlo simulation were used to generate these plots. See text for details.

associated with the shortest latency period can be increased. The general tendency is that the greater the number of teams, the smaller is the critical initial density in favor of the opinion associated with the shortest mean latency period (see Figures 7(a), 7(c), and 7(e)). However, as k approaches $N/3$ the system stops obeying this tendency. This is a second result of this experiment. When $k = N/3$ the system does not achieve consensus (curve plotted in gray). Although counterintuitive at first sight, the reason for this result is simple. When $k = N/3$, every time a team becomes non-latent, a new team is formed but with exactly the same members because the probability of having two teams leaving the latent state at exactly the same time is zero. This means that when $k = N/3$ there is no change in the density of opinions after the initial team formations.

In terms of the number of team formations to achieve consensus, the results are not surprising. In Figures 7(b), 7(d), and 7(f) it is shown how as the number of teams increases, so does the number of team formations needed to reach consensus. As before, the peak of these curves is reached at the critical initial density.

Note that in all cases, the dynamics are better observed when the population size is relatively large. When the population is small, the system is so low in resolution that it is difficult to observe any effect.

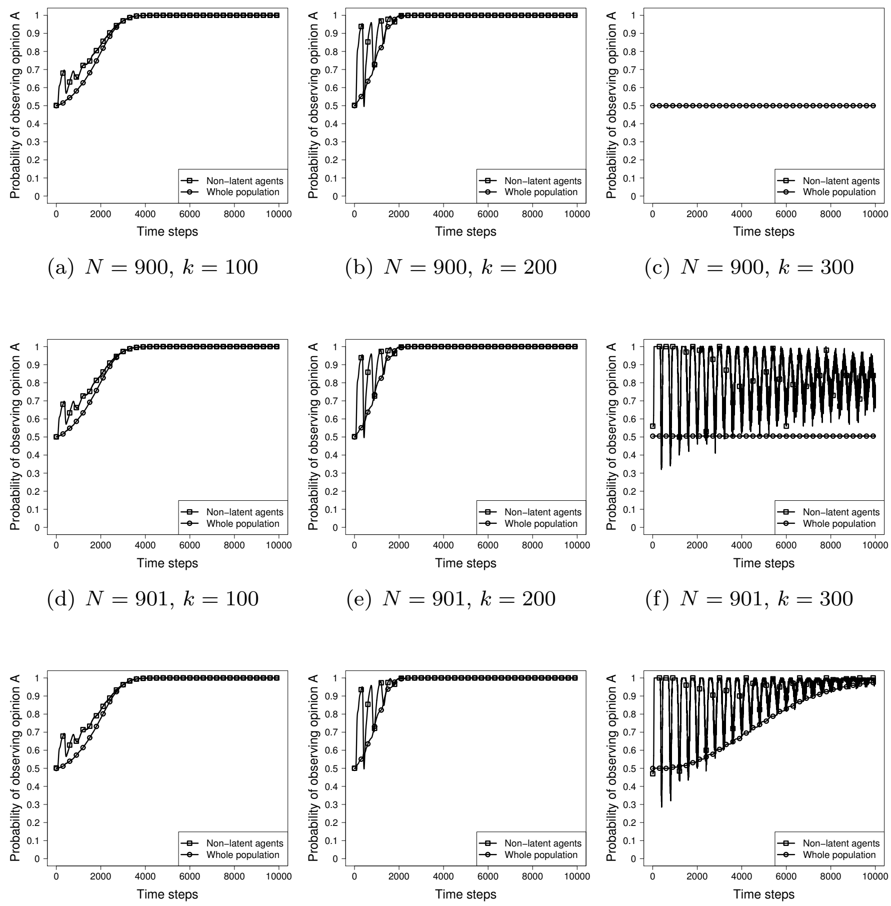
3.3.2 Experiment 2: Development over Time

A second experiment was carried out to understand the dynamics of the system as the number of teams approaches the limit $N/3$ and to explain why the system exhibits different critical initial densities when different latency duration ratios or different number of teams are used. Unless otherwise stated, we used the same setup used for the previous experiment.

The experiment consists in tracking over time the number of latent and non-latent agents with the opinion associated with the shortest latency period (opinion A). With these numbers, we compute the total proportion of agents with opinion A, and the proportion of non-latent agents with opinion A. The first piece of information tells us the state of the system. The second piece of information is the proportion of non-latent agents that "advertise" opinion A when a team leaves the latent state.

In Figure 8, we show the development of these proportions over time when the population size, and the number of teams vary. For these plots, we fixed $r = 4$, and the initial density of agents with opinion A to 0.5. The first row of plots corresponds to a total number of agents equal to 900. The second and third rows correspond to 901 and 902 agents respectively. From left to right, the columns show the plots that correspond to 100, 200, and 300 teams respectively.

In the previous experiment, we saw that when N, the population size, is a multiple of $3k$, that is, when all agents are in a latent state at any point in time, the system does not exhibit any dynamics. This phenomenon can be seen in Figure 8(c). In this experiment, we observe that when $N = 3k + 1$ (see Figure 8(f)) there is some dynamics in the non-latent subpopulation. However, there is no dynamics at the level of the whole population. Thus, when $N = 3k$ and $N = 3k + 1$, the population of agents does not achieve consensus. The dynamics that we observe in the case $N = 3k + 1$ are present only in the non-latent subpopulation, which consists in this case of one single agent. This non-latent agent together with the members of any team that leaves the latent state create a subpopulation of four agents from which a new team is formed. However, because at least three of these agents have the same opinion, this agent has no possibility to change the opinion of the agents that just switched state. Thus, one single non-latent agent cannot induce population-wide dynamics and



(a) $N = 900$, $k = 100$ (b) $N = 900$, $k = 200$ (c) $N = 900$, $k = 300$

(d) $N = 901$, $k = 100$ (e) $N = 901$, $k = 200$ (f) $N = 901$, $k = 300$

(g) $N = 902$, $k = 100$ (h) $N = 902$, $k = 200$ (i) $N = 902$, $k = 300$

Figure 8: Development over time of the total proportion of agents with opinion A, and the proportion of non-latent agents with opinion A. Opinion A is associated with the shortest latency period. For these plots, the latency duration ratio is equal to four and the initial density of agents with opinion A is equal to 0.5. These plots are based on data gathered through 1,000 independent runs of a Monte Carlo simulation.

therefore, no consensus can be reached.

When the non-latent population is composed of two agents, the population can reach consensus (see Figure 8(f)). Two non-latent agents are enough for possibly changing the opinion of one agent that just switched from a latent to a non-latent state. Thus, a non-latent population of at least two non-latent agents guarantees consensus.

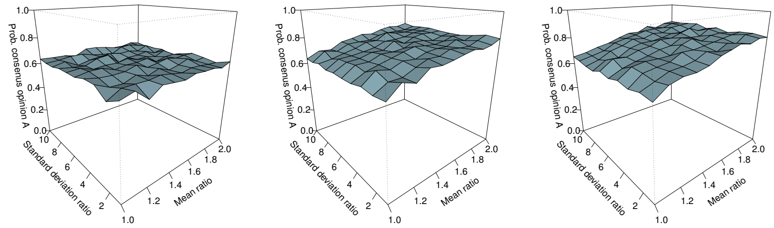
The proportion of individuals with the opinion associated with the shortest mean latency period evolves in an undulatory way. These "waves" are especially evident in the non-latent population. This phenomenon is caused by the existence of two different latency periods. In Figure 8, we can see how the valleys of the waves concur with multiples of the mean of the slowest latency period. In other words, the period of these waves is μ_B. In our example, $\mu_B = 400$ because $r = 4$ and $\mu_A = 100$. The amplitude of these waves is proportional to the number of teams of latent agents. This is simply a consequence of having more agents switching between latent and non-latent states.

The wave-like variations in the proportion of non-latent agents with one or the other opinion explain the existence of critical initial densities. They also explain their change in value when the latency duration ratio or the number of teams change. A latency duration ratio greater than one gives more time to teams with agents with opinion A than to teams with opinion B to accumulate agents with that same opinion in the non-latent subpopulation. Given that $\mu_B = r\mu_A$, by the time the first teams with opinion B leave the latent state, teams with opinion A will have switched states approximately r times. This imbalance makes the population reach consensus on opinion A with higher probability than on opinion B. If the initial density is such that at the beginning of the process there are more teams with opinion B than with opinion A, that is, the initial density is lower than 0.5, then it is possible to balance the system. In such a situation consensus will be reached on either of the two opinions but due to random fluctuations. Thus, the initial density that balances the opinion update process in the non-latent population is the initial critical density. A similar reasoning explains why the initial critical density decreases when the number of teams increases.

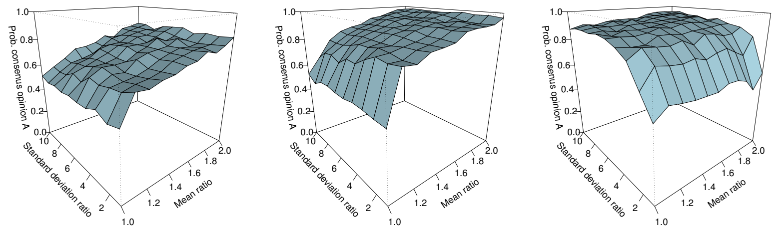
3.3.3 Experiment 3: Decision Power

If there is no *a priori* information about the latency duration distributions, there is no reasonable population initialization strategy other than with a density equal to 0.5. Under these circumstances we are interested in measuring the decision power of the system, that is, we would like to know with what probability the population of agents reaches consensus on the opinion associated with the shortest latency duration. To meet this goal we fixed the parameters of the distribution associated with the shortest latency period (μ_A, σ_A). Then, we varied both the mean and standard deviation of the distribution associated with the longest latency period (μ_B, σ_B). The explored ranges were: $\mu_B = r\mu_A$ with $r \in [1.0, 2.0]$ in increments of 0.1, and $\sigma_B = s\sigma_A$ with $s \in [1.0, 10.0]$ in increments of 1.0. This experiment was designed with the purpose of studying the behavior of the system with different levels of overlap between the two distributions. The parameters used for the distribution associated with the shortest latency period were $\mu_A = 100$, and $\sigma = 2$. Other values were explored but the system did not exhibit different dynamics. The analysis is valid as long as the relations between the distributions' coefficients of variation remain the same. To ensure consensus, we increased the population size with respect to the previous experiments by adding two non-latent agents. The results are shown in Figure 9.

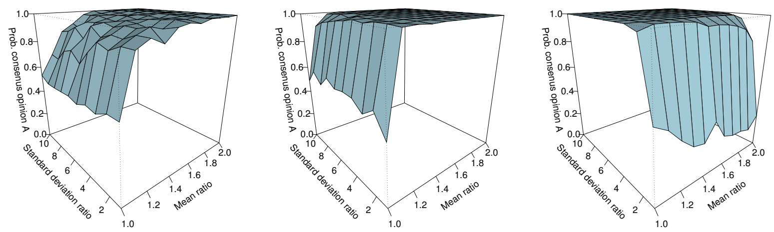
With 11 agents (Figures 9(a), 9(b), and 9(c)), the minimum probability is approximately equal to 0.6. With one team, we know that the population achieves consensus but on a random opinion. Thus, the probability does not change at any point with one team. With two and three teams, the probability increases with the ratio between the means until reaching a value of approximately 0.8. The ratio between standard deviations does not seem to affect the



(a) $N = 11$, $k = 1$ (b) $N = 11$, $k = 2$ (c) $N = 11$, $k = 3$



(d) $N = 92$, $k = 10$ (e) $N = 92$, $k = 20$ (f) $N = 92$, $k = 30$



(g) $N = 902$, $k = 100$ (h) $N = 902$, $k = 200$ (i) $N = 902$, $k = 300$

Figure 9: Probability of reaching consensus on the opinion associated with the shortest latency period as a function of different level of overlap between latency duration distributions. The mean of the longest latency period, μ_B, is computed as $r\mu_A$, where μ_A is the mean of the shortest latency period (equal to 100 time steps), and $r \in [1.0, 2.0]$ in increments of 0.1. Likewise, $\sigma_B = s\sigma_A$ with $\sigma_A = 2$, and $s \in [1.0, 10.0]$ in increments of 1.0. The initial density is equal to 0.5 and the results are based on data gathered through 1,000 independent runs of a Monte Carlo simulation.

system. With 92 agents and 10 teams (Figure 9(d)) the probability increases from a value of approximately 0.5 at $r = 1$ to a value of 0.8 at $r = 2$. With 20 teams (Figure 9(e)), the probability increases from 0.5 at $r = 1$ to a value greater than 0.95 at $r = 2$. In both cases, it is the ratio between means what causes the increment of the probability value. The ratio between standard deviations does not seem to affect the system. However, with 92 agents and 30 teams (Figure 9(f)) the ratio between standard deviations does have an impact on the system. The probability increases from a value of 0.5 at $s = 1$ to a value of approximately 0.85 at $s > 5$. In this case, the ratio between means does not affect the system. The same high-level behavior can be observed with 902 agents. With 100 and 200 teams (Figures 9(g) and 9(h)), the system is mostly affected by the ratio between means. When using 200 teams, the system reaches a probability of one to achieve consensus on the opinion associated with the shortest latency period already from $r \geq 1.2$. With 300 agents (Figure 9(i)), it is mostly the ratio between the standard deviations what affects the system, reaching a probability of one from $s \geq 3$.

In summary, the system is able to detect the opinion associated with the shortest latency period under a wide range of combinations of means and standard deviations of the distributions involved. The probability of reaching consensus on the opinion associated with the shortest latency period grows more rapidly when a large number of teams, and consequently, a large population is used. Distributions with high levels of overlap are successfully distinguished. When the number of teams approaches the limit $N/3$ the system exhibits interesting dynamics that can be used to detect the opinion associated with the latency period with the lowest standard deviation.

4 Self-Organized Collective Decision-Making in Swarms of Robots

In this section, we demonstrate an example application of the proposed model's dynamics as a self-organized collective decision-making mechanism for robot swarms. First, we discuss why we consider such a mechanism to be self-organized. Then, we describe the robotics task that serves as an example of the kind of situations the proposed mechanism could be used for. We conclude by presenting results that confirm the effectiveness of the approach.

4.1 Majority-Rule Opinion Dynamics with Differential Latency and Self-Organization

The majority-rule opinion dynamics with differential latency can be said to be an example of a self-organization process. The reasons are the following. First, a large-scale pattern emerges as the result of purely local interactions among the system constituent entities. In our model, the large-scale pattern is consensus and it arises from many team-level interactions. Second, the rule that agents follow to exchange and integrate information, that is, the majority rule, does not make any reference to the population-level pattern that emerges. Third, no single agent is capable of supervising or controlling the evolution of the system because agents have knowledge only about their own and their team members' opinions. There is no obvious way for them to know whether the population has reached a consensus or not. Finally, the proposed model's dynamics adhere to the four principles of self-organization (as described by Camazine et al. (2003); Moussaid et al. (2009)):

1. Positive feedback. In a differential latency scenario, agents whose opinion is associated with the shortest latency period leave the latent state before others. Thus, the probability that a new team has a majority in favor of

the opinion that these agents had increases. If the latency duration ratio is large enough, this process can be repeated several times before the first agents with the alternative opinion leave the latent state. The result is an imbalance of opinion representation in the non-latent subpopulation from which new teams are formed. Since this imbalance grows over time, the population eventually reaches total consensus.

2. Negative feedback. The positive feedback process described above can be counterbalanced if the number of latent agents with the opinion associated with the longest latency period is large enough. This occurs when the system begins with an initial density lower than or equal to the critical density as explained in Section 3.3.2. Another limiting factor that balances the system is the finiteness of the population. In other words, the system eventually reaches a stable state because the population has reached a consensus.

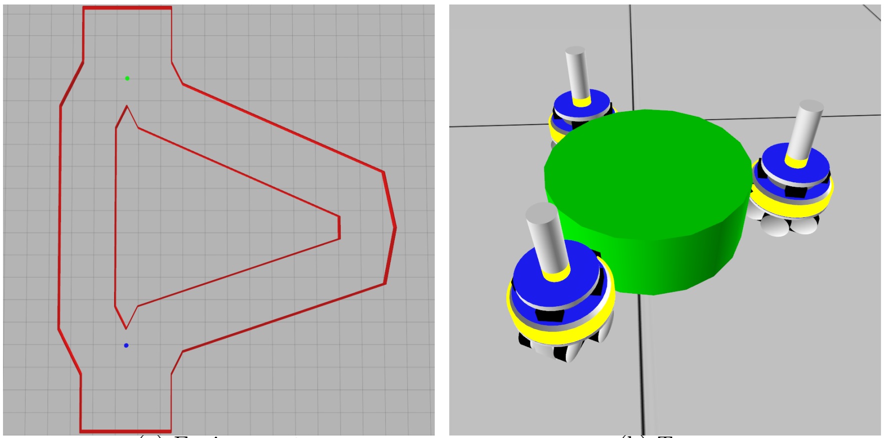
3. Amplification of fluctuations. Because teams are formed at random, the proportion of non-latent agents with one opinion or another may fluctuate randomly. These fluctuations can be amplified by the positive feedback process described in point 1. When the system is in an unstable state due to an initialization at the critical density, fluctuations allow the system to break the symmetry of the update "waves" we saw in Figure 8 and reach consensus on any of the two opinions available in the system.

4. Multiple direct or indirect interactions. In the model we are proposing, agents interact directly every time a new team is formed. Multiplicity is satisfied because teams must be destroyed and created many times before the population reaches a consensus.

4.2 Example Task: Foraging with Collective Transport

Goss et al. (1989) studied the collective decision-making capabilities of colonies of *Iridomyrmex humilis* ants. To make ants choose between two options, they connected the ants' nest with a food source by a bridge with two branches that differed in length. Their experiments showed that a colony of this kind of ants was able to choose more frequently the shortest branch of the connecting bridge. Ants can do that through pheromones, which are chemicals ants lay on the ground. Differences in the pheromones relative concentrations indicate the preferences of the colony. Ants detect these differences and reinforce the selected branch creating in this way a positive feedback process. This experiment has been a major source of inspiration to the swarm intelligence community. It was, for instance, the inspiration source for ant colony optimization (Dorigo, 2007; Dorigo and Stützle, 2004), which is one of the most successful optimization techniques derived from research in swarm intelligence.

We used a bridge-like environment similar to the one used by Goss et al. to study the decision-making capabilities of a robot swarm governed by the dynamics of the majority-rule opinion formation model with differential latency. An illustration of this environment is shown in Figure 10(a). The task of the robots is to transport objects from a starting location (located at the bottom part of the figure) to a goal location (located at the top part of the figure). The objects that need to be transported are heavier than the capacity of a single



(a) Environment (b) Team

Figure 10: Task Scenario. The arena is a bridge-like environment with two branches of different lengths (see Figure (a)). The task of the robots is to transport objects from the lower part to the upper part of the environment. Teams of robots start pre-attached to these objects (see Figure (b)). The choice robots must make is whether to take the left or the right path.

robot. Thus, teams of robots need to be assembled in order to move the objects around the environment. An assembled team ready to transport an object is shown in Figure 10(b). The choice robots have to make while performing this object transportation task is to go to the target location using either the left or the right path. These two options represent the robots' "opinions". The time it takes for a robot to go from the starting to the target location and back is the duration of the latency period associated with each opinion. In the context of swarm robotics, we refer to these latency periods as "action execution times" since the idea can be generalized to the time it takes for a robot to execute an action associated with an opinion.

In the experiments that will be described next, we used ARGOS (Pinciroli, 2007), a simulator especially developed for the *SWARMANOID* project.[4] The simulator uses the open dynamics engine library to accurately simulate physical interactions. The robot models are based on the physical and electronic designs of the actual *SWARMANOID* foot-bots (currently under development).

4.3 Experiment 1: Action Execution Time Distributions

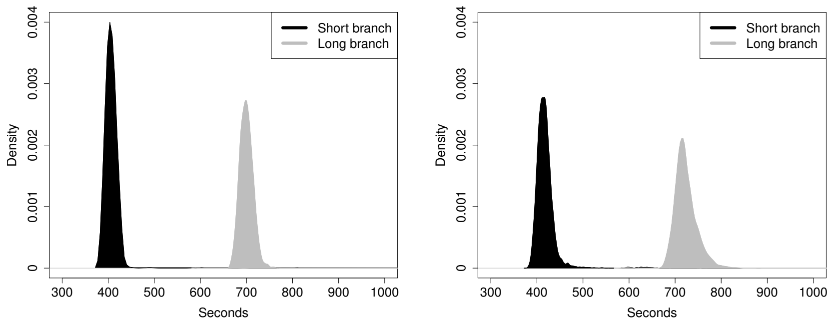
We conducted a first experiment to estimate the distributions of the action execution times associated with each branch of our bridge-like environment. To simulate the consensus on one of the branches, we initialized robots to always choose one of the two branches. We did this for both branches. In this experiment, we varied the number of teams to see how the tendency of the system to reach consensus affects these distributions.

The simulation proceeds as follows: Three robots are generated simultaneously as a team. These robots are physically pre-attached to the object to

[4] `http://www.swarmanoid.org/`

be carried through an on-board gripper actuator (see Figure 10(b)). Only one team is generated at a time. The next team is generated after 40 simulated seconds in order to avoid collisions. The process is repeated until the maximum number of teams have been deployed. This number is mostly determined by the dimensions of the environment. In this experiment, the environment can hold up to ten teams. Every time a team is generated, robots apply the majority rule to make a local decision on the path to follow. This is done by exchanging messages using the range and bearing communication device the robots are equipped with. Once robots agree on the path to follow, they execute the collective transport controller described in (Ferrante et al., 2010). This controller allows the robots to transport the object to the goal location while avoiding obstacles. The goal location is indicated by a light source located far above it, which the robots can perceive through their light sensors. Obstacles are detected using a rotating distance scanner, that is, a rotating infra-red emitter and receiver. To coordinate the heading direction, robots use the algorithm described in (Ferrante et al., 2010), which uses the range and bearing communication device. Finally, two LEDs are placed close to the junctions to let robot teams know in which direction they should turn. When close to a junction, robots use their omni-directional camera to detect the junction LED, which they use to turn left or right according to the chosen path. When robots reach the goal area they detach from the object they were transporting and go back through the chosen path as single robots. On their way back, robots try to go away from the light at the goal location using the same path they used when they were part of a team. However, because other teams may be close by and robots need to avoid obstacles, it can happen that individual robots have to go back to the starting location through the "wrong" path.

Example results of this experiment are shown in Figure 11. This figure shows the estimated distributions for the cases when there are two and ten teams in the environment. The three most important things to notice are: *a)* In all cases, the distributions are well separated, which means that there is a clear difference in terms of travel times between the two branches of the environment. The action execution time ratio and standard deviation ratio for the two-teams case are respectively (1.71,0.26) and (1.72, 1.23) for the ten-teams case. *b)* The distributions are right-skewed. This means that some robots take much longer than the average time required to go back to the starting location. Visual inspection of the robots' behavior revealed that this occurs when robots need to choose a path different from the one they used to reach the target location. As we mentioned above, this happens because robots avoid incoming teams and as a result, these robots find themselves in a path different from the one they originally chose to traverse. As expected, this phenomenon occurs more frequently when the number of teams in the environment increases. *c)* The mean and the variability of the distributions increase with the number of teams deployed in the environment. This is a phenomenon related to what we said in the previous point. As the number of teams in the environment increases, so does the level of interference between teams and robots. In other words, robots need to avoid more obstacles and move more slowly when the path they choose is filled near to capacity. Thus, we expect that while the swarm is operating, the action execution time distributions will change over time. This occurs as a direct result of the system's tendency of reaching a consensus on one of the two available opinions.



(a) Two teams

(b) Ten teams

Figure 11: Action execution time distributions. Figure (a) shows the estimated densities for the two available actions when there are two teams in the environment. Figure (b) shows the estimated densities when there are ten teams in the environment. Each density plot is based on 10,000 round trips (100 runs of 100 trips each) of a robot between the starting and goal locations in the environment shown in Figure 10(a).

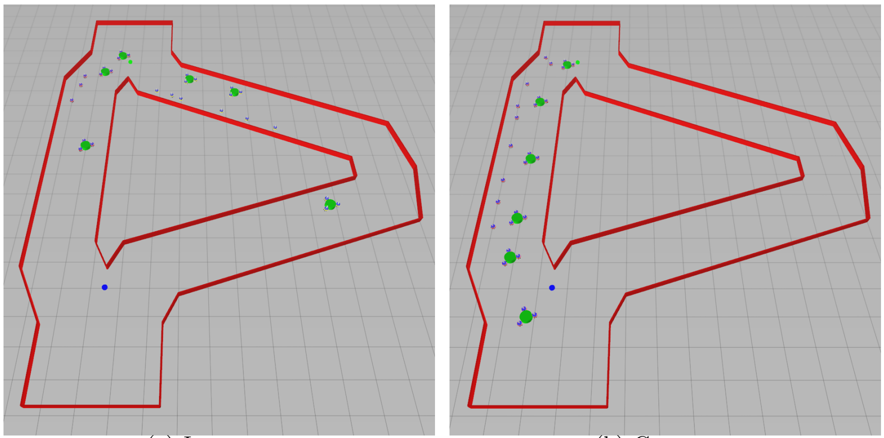
4.4 Experiment 2: Collective Decision-Making

In this second experiment, we measure the probability of a population of robots to choose the shortest path by consensus between the starting and goal locations in the environment shown in Figure 10(a). We also measure the number of team formations needed to reach consensus. With this experiment we test our hypothesis that states that the dynamics of the majority-rule opinion formation model with differential latency can be used as a self-organized collective decision-making mechanism for swarms of robots.

In this experiment, we varied the number of teams in the environment while keeping the population of robots constant. We used 32 robots in total. This choice is due to the maximum capacity of the environment, which can hold up to ten teams. We added two more robots in order to ensure consensus as explained in Section 3. The simulation works in exactly the same way as in the first experiment. The initial density of robots with the opinion associated with the shortest path is 0.5, that is, 16 robots initially favored the shortest path and 16 favored the longest one. In Figure 12, we show two snapshots of a simulation run that finishes with the swarm selecting the shortest path between the starting and target locations.[5]

The results of this experiment are summarized in Table 2. This table shows the estimated probabilities of the swarm choosing the shortest branch of the environment as a function of the number of teams. These data are based on statistics take from 100 independent simulation runs. To compare results with what our model would predict, we also ran our Monte Carlo simulation using the data gathered in the first experiment. Specifically, we set the mean and standard deviation of the latency period associated with the shortest path to

[5] At http://iridia.ulb.ac.be/supp/IridiaSupp2010-014/, the reader can find video clips that show the collective decision-making mechanism in action.



(a) In progress (b) Consensus

Figure 12: Shortest path selection process. Figure (a) shows a swarm of robots in the process of transporting objects from the starting to the target location. At this stage, the swarm has not reached consensus yet and thus robots still use both branches of the environment. Figure (b) shows the state of the environment when the a swarm of robots has reached consensus. The path selected by the swarm of robots is the shortest one.

100 and 20 time steps, respectively. The mean of the latency period associated with the longest path was set to $1.72 \times 100 = 172$ time steps, and its standard deviation was set to $\lceil 1.23 \times 20 \rceil = 25$ time steps.

Table 2: Probability of choosing the shortest branch of the environment as a function of the number of teams k. The population size N is equal to 32 robots.

	Physics-Based Simulation		Monte Carlo Simulation	
k	Probability	Avg. Team Formations	Probability	Avg. Team Formations
1	0.48	74.29	0.54	70.66
2	0.52	72.67	0.62	74.62
3	0.69	72.75	0.58	74.39
4	0.71	70.28	0.68	71.87
5	0.75	71.60	0.74	70.17
6	0.74	75.22	0.72	71.18
7	0.79	76.20	0.83	80.84
8	**0.86**	77.73	0.82	85.58
9	0.83	81.29	**0.86**	98.43
10	0.81	109.95	0.69	248.25

The probability of choosing the shortest path increases with the number of teams up to a certain limit. In fact, the maximum probability is reached with eight teams in the case of the physics-based simulator and with nine with Monte Carlo simulation. In both cases, the maximum probability found was equal to 0.86. The average team formations needed to reach consensus oscillates within the range $[70, 75]$ for most cases and grows substantially when the number of teams approaches the limit $N/3$, where N is the number of robots. To explain this result, we should recall the analysis presented in Section 3. First, the action execution time ratio is lower than two. This condition proved to be difficult for small swarms (our 32-robot swarm can still be considered to be small). Second, as the number of teams approaches the limit $N/3$, the size of the non-latent subpopulation starts playing a role in both the quality of the decision eventually made by the swarm (lowering its quality) and the time it takes to reach consensus (increasing the number of needed team formations). The difference in the results obtained with the physics-based simulator and our Monte Carlo simulation, specially regarding the number of team formations needed to reach consensus, may be due to the way teams are treated. In the physics-based simulator, a 40 seconds long delay precedes each team formation. This delay makes it possible for robots to meet at the starting location after transporting an object. This enables teams that reach the starting location within a time difference lower than 40 seconds to exchange individuals. This phenomenon never occurs in the model and our simulations presented in Section 3. Thus, it seems that deploying teams sequentially reduces the number of team formations needed to reach consensus when the number of teams reaches the limit $N/3$.

In summary, in the first experiment we have seen that the action execution time distribution associated with an opinion is not independent of the swarm of robots or the environment. In fact, we have seen that it depends on the number of active robots that at some point in time have that opinion. As the swarm of robots reaches consensus, the distributions associated with each opinion change. In the second experiment, we measured the probability of the swarm selecting the shortest branch of our bridge-like environment. Under the tested circumstances, a swarm of 32 robots selects the shortest path between a starting and target locations with a probability of 0.86 in less than 100 team formations. Based on the analysis of the previous section, we can expect that this probabilty increases if the action execution time ratio increases. This can occur if the environment is larger, for example. These results show that the proposed approach is affective as a self-organized collective decision-making mechanism for swarms of robots.

5 Discussion

In this section, we put our proposal into the proper context by pinpointing the differences and similarities of our approach with works described in the specialized literature. After that, we discuss some limitations of the current proposal and possible ways of overcoming them.

5.1 Collective Decision-Making in Artificial Swarms

As we mentioned in the introduction, swarm intelligence researchers have always tried to endow artificial swarms with decision-making capabilities similar to the ones of natural swarms. Although the swarm intelligence field, as we now know it, was born within the broader field of robotics (Beni, 2005), the first truly successful attempts to do this occurred within the realm of software. For example, in ant colony optimization algorithms (Dorigo and Stützle, 2004), a population of agents, loosely mimicking the pheromone-laying and pheromone-following behavior of real ants, collectively select good solutions to hard optimization problems from a very large set of candidates. In robotics, collective decision-making is harder to achieve than in software because the environment is mostly unknown.

Many collective decision-making mechanisms in swarm robotics have also been inspired by the behavior of insects, and in particular of ants. For instance, a very important category of approaches is based on the simulation of pheromones. Approaches in this category range from the use of real chemicals (Fujisawa et al., 2008a,b; Russell, 1999), to the use of digital video projectors to draw pheromone trails on the ground (Garnier et al., 2007; Hamman et al., 2007; Sugawara et al., 2004). This category of approaches also include those in which the environment is enhanced in order to let it give robots information that is normally encoded in the concentration of pheromones. For example, Mamei and Zambonelli (2005) and Herianto and Kurabayashi (2009) deploy RFID tags in the environment so that robots can read from or write in them. In (Mayet et al., 2010), the authors use an environment whose floor is covered with a paint that glows if ultraviolet light is applied to it. The authors simulate pheromones by making robots activate ultraviolet LEDs as they move. A further variant of this approach is the use of actual robots as markers to form trails. Some works that use this approach are (Ducatelle et al., 2010; Nouyan et al., 2008, 2009; Payton et al., 2001; Werger and Matarić, 1996). Simulating pheromones, at least in the way it has been done so far, has important limitations. For example, dealing with chemicals is problematic because robots need very specialized sensors. Because designing such sensors is not an easy task, some authors have even tried using antennae of real insects (Kuwana et al., 1995; Nagasawa et al., 1999). Using video projectors is an approach that can be used only indoors and under controlled conditions. Furthermore, the use of video projectors implies the use of tracking cameras and a central computer to generate the images to be projected. The existence of such a central information processing unit makes the approach depart from swarm intelligence principles. Modifying the environment with special floors or with RFID tags is a cheap and interesting approach. However, their applicability is limited to situation in which it is possible to have access to the environment before robots are deployed. Finally, using robots as markers allows a swarm to operate in unknown environments and no central control is required. However, complex robot controllers are needed in order to allow individual robots to play different roles in the swarm. While a promising approach, the development of complex robot control software for swarms is still in early stages of research as we are still trying to understand the connection between individual-level and collective-level behaviors.

The mechanism that we are proposing in this article does not rely on the simulation of pheromones. However, one could find connections between the two approaches. First, one could interpret the population as a distributed pheromone model. For example, the proportion of agents with opinion A represents the pheromone level associated with that opinion. Second, the majority rule simultaneously reinforces and evaporates the pheromone levels of the two opinions. This occurs when the majority rule makes an agent change its opinion. One opinion gains one agent, while the other simultaneously loses one agent. Finally, because teams are formed at random, it is possible to have a very noisy idea of the opinion that has the actual majority in the whole population. For example, if there is no majority, it is likely that the first two agents of a team that is being formed do not share the same opinion. In this case, the third agent would decide the opinion the team will adopt. In contrast, if there is a majority, the likelihood of those two agents having the same opinion grows and the opinion of the third agent becomes irrelevant.

While the pheromone-laying and pheromone-following behavior of ants has inspired most collective-decision making mechanisms for artificial swarms, other insect behaviors have also served as inspiration sources. Trophallaxis, the exchange of liquid food between insects, has been the inspiring phenomenon behind the proposal of Gutiérrez et al. (2010), who propose a method through which a swarm of robots can locate and navigate to the closest location of interest from a particular origin. The method Gutiérrez et al. propose needs robots to implicitly know what they are supposed to achieve. Robots measure the distance they have traveled and communicate this information to other robots in order to reduce the uncertainty of each robot's estimate of the location of a target. In our work, robots do not measure neither travel times nor distances and still, the swarm finds the shortest path between two locations.

Another insect behavior that has triggered research in collective decision-making in swarms of robots is that of aggregation and quorum sensing. For example, the aggregation behavior of cockroaches has been the source of inspiration for a site-selection mechanism with robots (Garnier et al., 2009). The nest-selection mechanism use by ants, which is based on detecting a quorum in favor of one option, has inspired the work of Parker and Zhang (Parker and Zhang, 2009, 2010). In these works, robots need to know whether there are enough committed robots for one of the competing options. In both cases, the more committed robots there are for one of the options, the more likely it is for a robot to commit for that option too. In Garnier et al.'s work the decision is probabilistic, and in Parker and Zhang's work the decision depends on whether the number of committed robots is larger than a threshold. Deciding the value of this threshold or the rate at which the commitment probability increases is a critical issue because the first alternative that is identified as dominant will be the alternative chosen by the swarm. In our work, there are no thresholds or probabilities that depend on the number of robots with one opinion or the other. Thus, decision-making is a continuous process until the whole population reaches a consensus.

Finally, the work of Wessnitzer and Melhuish (2003) is related to ours too in the sense that they use the majority rule to make robots make decisions. Through the majority rule, robots decide which of two "prey" to chase and immobilize. Robots capture one prey after the other. Although the decision is collective, the majority rule is used simply to break the symmetry of the decision problem.

5.2 Limitations

We believe that the proposed collective decision-making mechanism is promising given that it enables a swarm of robots to make a decision that from an observer's point of view is intelligent without requiring intelligent individual decision makers. However, we are aware that the approach has the following limitations:

- Robots know exactly when they need to form teams and make a local decision. As we said before, one of the main problems in robotics is that the environment is unknown. Thus, it is difficult for a robot to know where it is and therefore when it should form a team. In our experiments, teams were pre-assembled in order to avoid this problem. Additionally, we used LEDs to let robots know when and how to turn. Future work should address this issue.

- Robots know the number of available alternatives. In this paper we focused on the case where there are two opinions only. It is easy to imagine an extension of the model in order to make it deal with more opinions. However, the main limitation of this and of such an extension is that robots know how many opinions there are. A possible approach to deal with this issue is to use some form of learning whereby robots can add opinions to the system as they interact with the environment.

- Consensus is a good solution only in some cases. It is easy to imagine a situation in which the optimal allocation of robots is 80% for one opinion and 20% for the other one. In such cases, consensus on one of the two opinions will be a suboptimal solution. Integrating opinion dynamics with task allocation methods could be a possible solution to this problem.

- If the environment changes after the system crossed has reached a consensus, the population cannot adapt. This problem could be tackled if some fixed number of robots do not change opinion. Ongoing work is already exploring this idea.

- The opinion dynamics that allows the swarm to reach consensus on one opinion is based on time-related "rewards". This means that the faster action is rewarded implicitly by giving robots that choose this action the chance to spread their opinion faster than other robots. In many situations, the desirable action may not be constrained by time. Therefore, ways of extending the proposed decision-making mechanism in order to make it deal with qualitative aspects are needed. A first approach to deal with this issue would be to translate into time these qualitative aspects of the alternative actions the swarm needs to choose from. For example, if some resource is more preferable to other, robots in favor of that resource should spend less time as non-latent than the other robots. In this way, a positive feedback process could favor that option. However, other methods should be explored.

- The decision quality depends on the population size. We saw that the larger the size of the population, the better is the decision made by the swarm. While this is related to scalability, which is a desirable property of swarm robotics systems, it hinders its use in real robotics systems. The reason for this is that, at least until now, the promise of having thousands of cheap robots has not been met. Research is needed in order to improve the decision quality of the swarm when its size is relatively small. An option, currently under study, is to simulate large swarms by making robots remember their past over long time horizons (not of just one action execution as it is currently done) and make a decision based on the opinion that has been observed more often during that period.

6 Conclusions

In this paper, we have introduced the concept of *differential latency* to the majority-rule opinion formation model. This model operates on a population of agents that can be in one of two states, called opinions. Teams of agents are repeatedly formed from randomly picked agents and the majority rule is applied locally. The result of this process is consensus, a collective state in which all agents have the same opinion. Latency is a period of time of stochastic duration during which agents cannot be influenced by other agents. When latency is used in the majority-rule opinion formation model, agents that switch opinion go into a latent state if they were not already in it. If agents were already in a latent state, they switch back to a non-latent state with a certain probability. The result is that the system may or may not reach consensus depending on the value of this probability. Differential latency is a new concept in the context of opinion formation models. Differential latency means that when agents go into a latent state, they remain in that state for a duration that depends on the opinion they just adopted. We show in this paper that when the durations of these latency periods are different, the population is more likely to reach consensus on the opinion associated with the shortest latency period. We demonstrated that this is the case for latency periods that are exponentially and normally distributed.

The opinion dynamics of the majority-rule opinion formation model with differential latency can be exploited in the field of swarm robotics as a self-organized collective decision-making mechanism. In a swarm robotics setting, agents are robots, opinions are actions, and latency periods are the duration of an action execution. We demonstrated the potential of the new approach as a collective decision-making mechanism in a scenario based on the well-known double bridge experiment that allowed researchers to show that ant colonies are able to find the shortest path between their nest and a food source. The results of our experiment clearly show that through the proposed mechanism a swarm of robots is able to find the shortest path between two locations without simulating pheromones or requiring robots to measure travel times or distances.

Collective decision-making in artificial swarms has been inspired by insect behaviors since the birth of swarm intelligence as a research field. In this paper we do not take inspiration from any natural phenomenon. We used instead a purely abstract model that exhibits interesting dynamics when coupled with features of real swarm intelligence systems. Our results suggest that it may be time to start departing from the field's historical inspiration source. From the engineering point of view this is not a problem because the interest there is to build effective systems. From the biological point of view, we believe that opinion formation models may be another tool to understand the underlying physical properties of animals' collective decision-making mechanisms.

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