A world map with a blue and white color scheme, showing the continents and oceans. The map is centered on the Atlantic Ocean and is slightly faded, serving as a background for the text.

What is Scaling? And why is it everywhere?

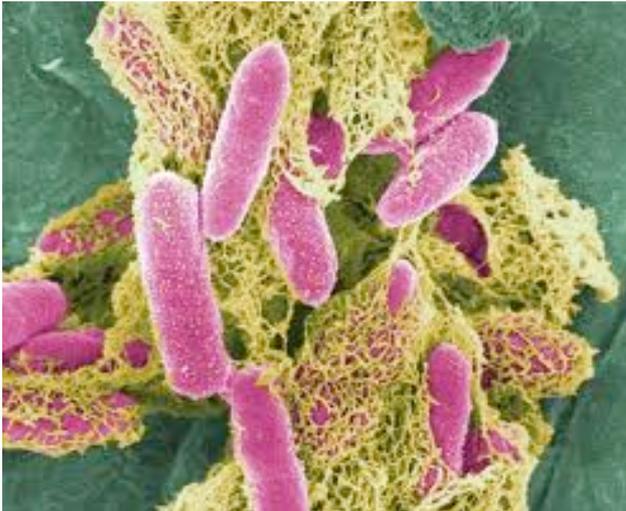
Melanie Moses
Department of Computer Science
University of New Mexico

Complex Adaptive Systems, CS 523
April 7, 2017

Organisms span an enormous range of sizes

A whale is
100 000 000 000 000 000 000 000
times bigger than an
E. coli

10^{-12} g



Glaw et al PLoS ONE 2012

10^8 g



Metabolic Scaling

A striking universal(?) pattern

Metabolic rate scales sub-linearly with mass

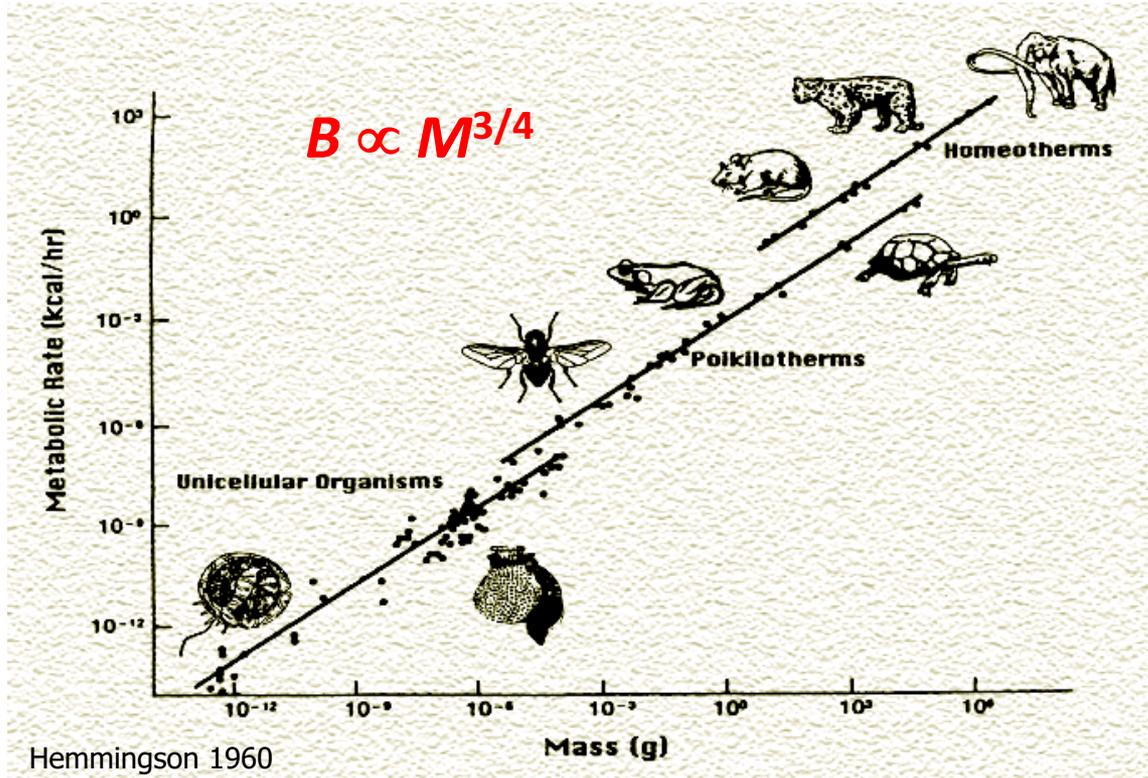
Metabolism is the rate of energy use

Measured as

Oxygen in, or CO₂ out

Food consumption

(for non-growing animals)



Metabolism governs

Physiology

Growth

Reproduction

Lifespan

Photosynthesis & carbon flux

Ecosystem dynamics...

Analyzing Scaling Relationships

$$B = cM^{3/4}$$

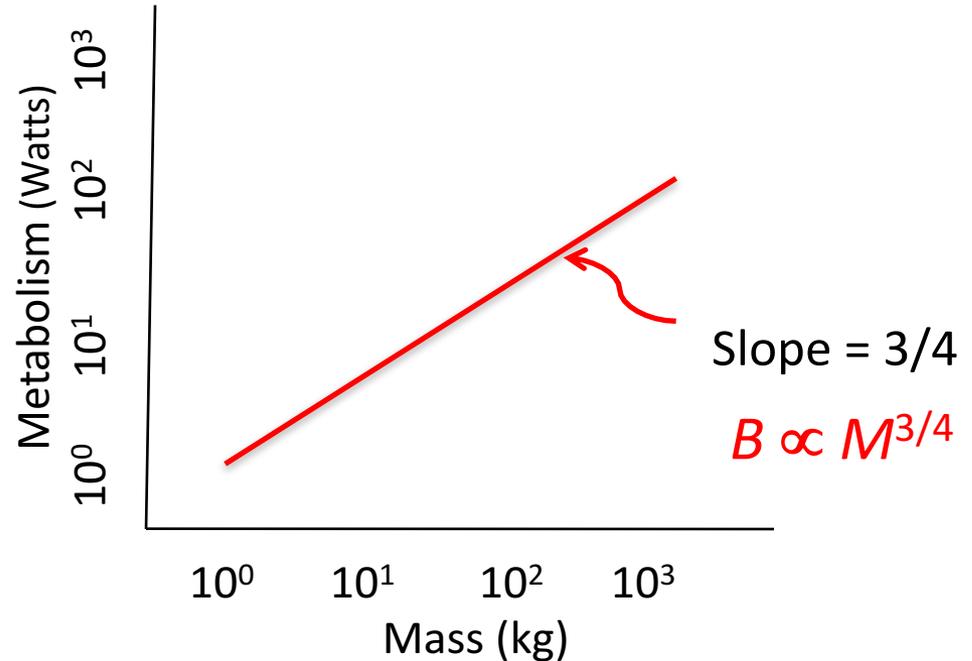
The scaling exponent is the slope
on log-log plot

$$\log(B) = \frac{3}{4} \log(M) + \log(c)$$

$$y = mx + b$$

The intercept is $\log(c)$

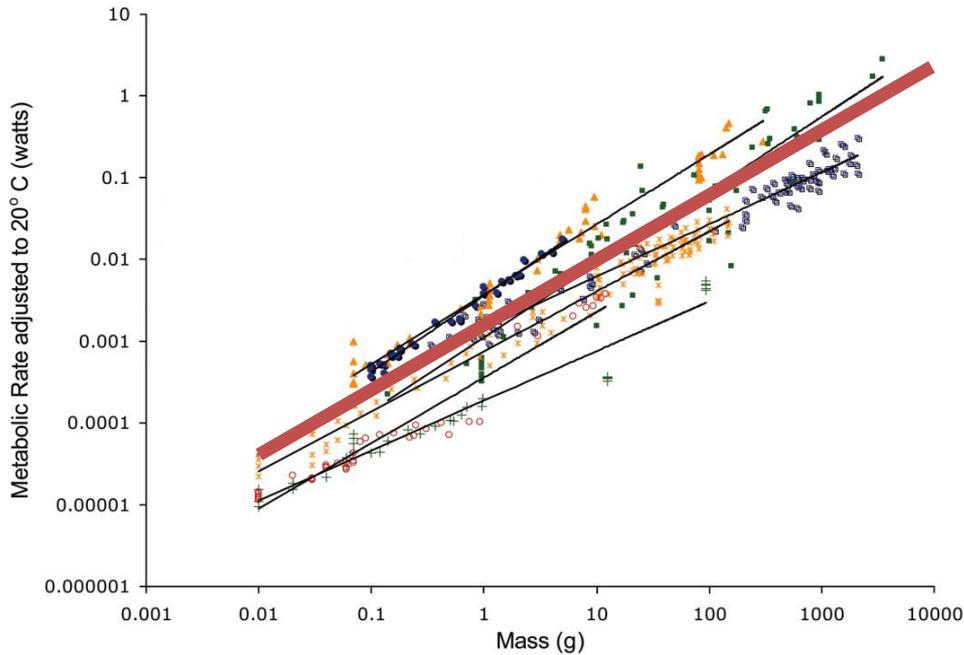
The slope is $\frac{3}{4}$



Meaningful variation around a mean of $\frac{3}{4}$

Metabolic Rate in Growing Fish

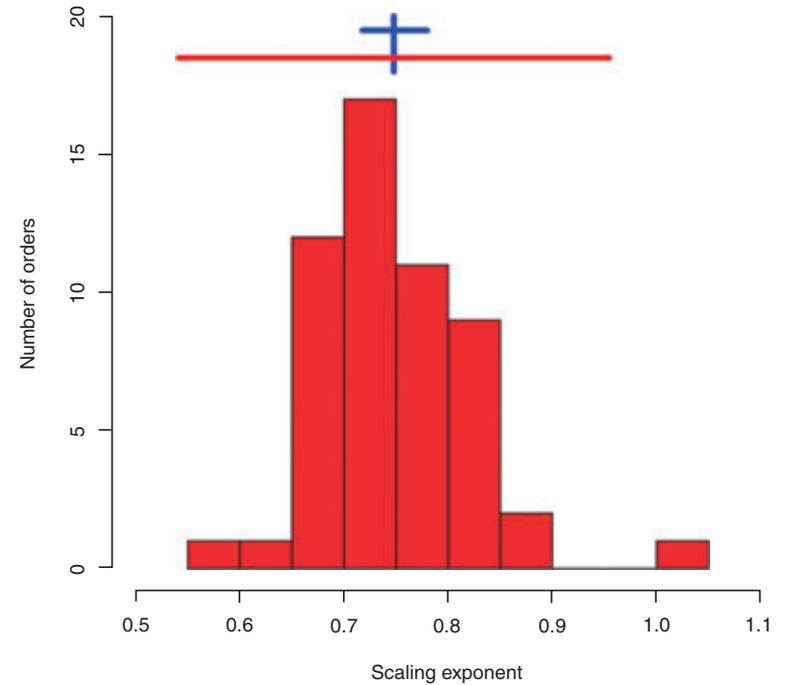
$$B \propto M^{3/4} \quad (0.6 - 0.9)$$



Moses et al AmNat 2008

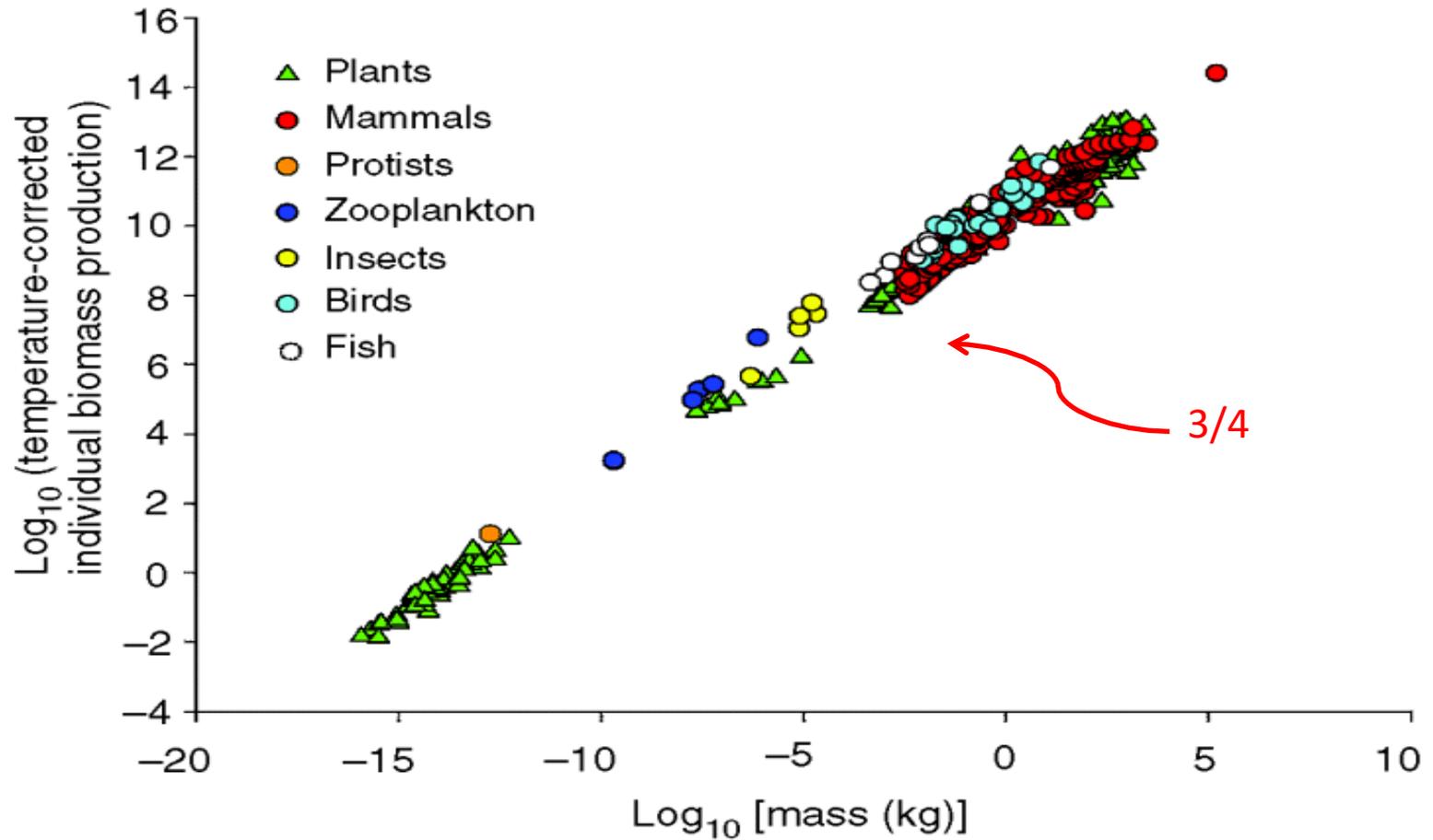
Metabolic Rate in Mammalian Orders

$$\text{Mean } B \propto M^{0.749} \quad (0.7 - 0.8)$$



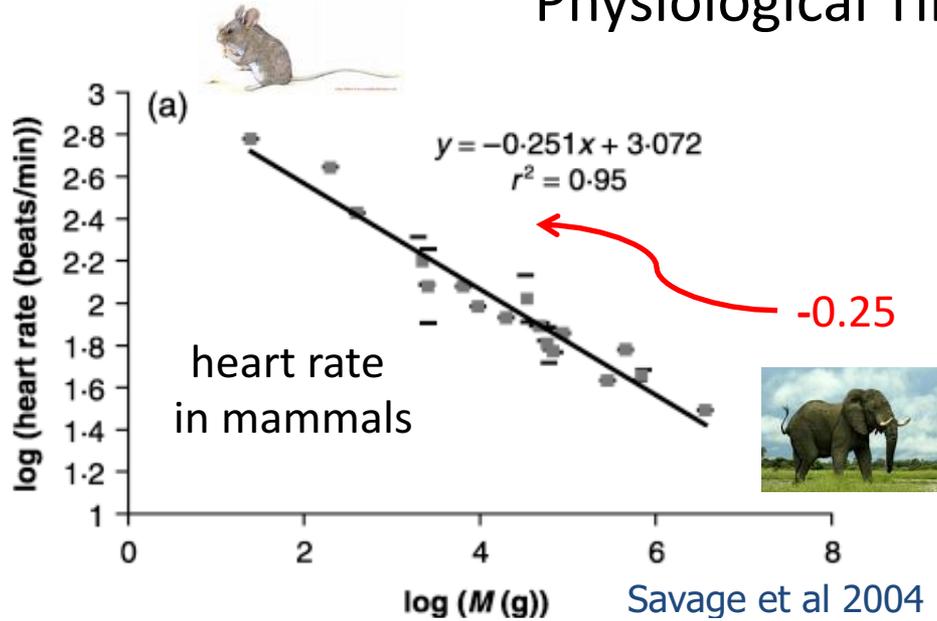
Isaac & Carbone Ecol Lett 2010

Biomass Production: $P \propto M^{3/4}$



Physiological Rates $\sim M^{-1/4}$

Physiological Times $\sim M^{1/4}$



Whole animal: $B \sim M^{3/4}$

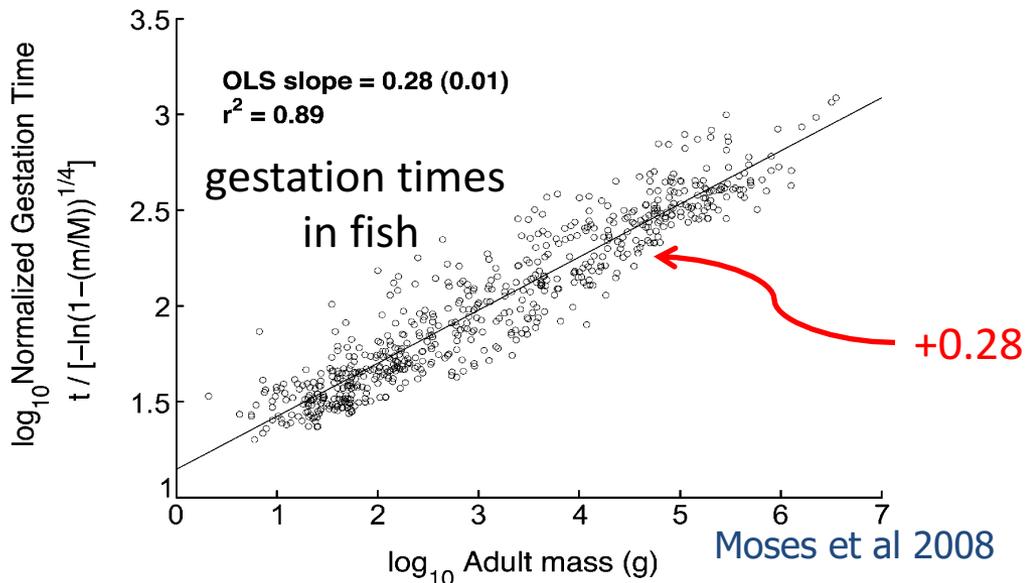
Mass-specific Scaling

$$\frac{B}{M} \propto M^{-1/4}$$

mass specific rates $\propto M^{-1/4}$

times $\propto M^{1/4}$

Mice live fast and die young



Universal Growth Curve

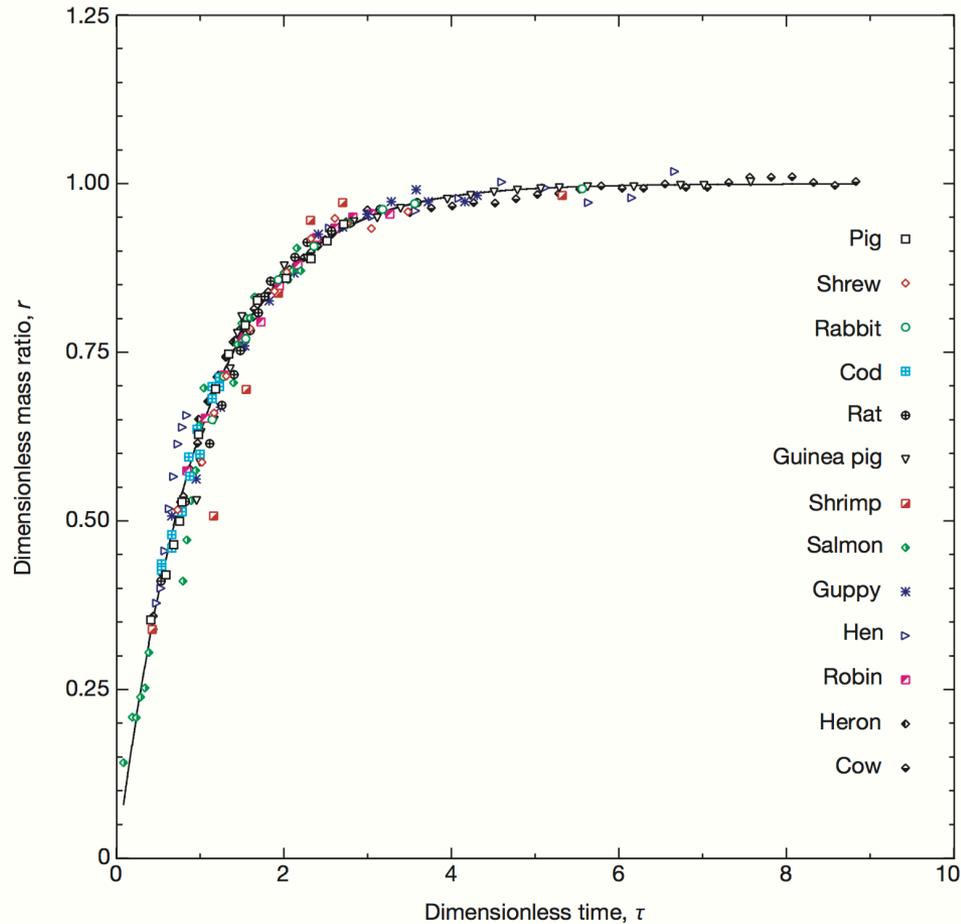


Figure 2 Universal growth curve. A plot of the dimensionless mass ratio, $r = 1 - R \equiv (m/M)^{1/4}$, versus the dimensionless time variable, $\tau = (at/4M^{1/4}) - \ln[1 - (m_0/M)^{1/4}]$, for a wide variety of determinate and indeterminate species. When plotted in this way, our model predicts that growth curves for all organisms should fall on

the same universal parameterless curve $1 - e^{-\tau}$ (shown as a solid line). The model identifies r as the proportion of total lifetime metabolic power used for maintenance and other activities.

How has evolution produced foraging strategies that scale up to millions of ants?

~20 Ants



© alexanderfeld.com

**Ants are
Abundant, Diverse, Dominant**

14,000 species

10^{19} ants

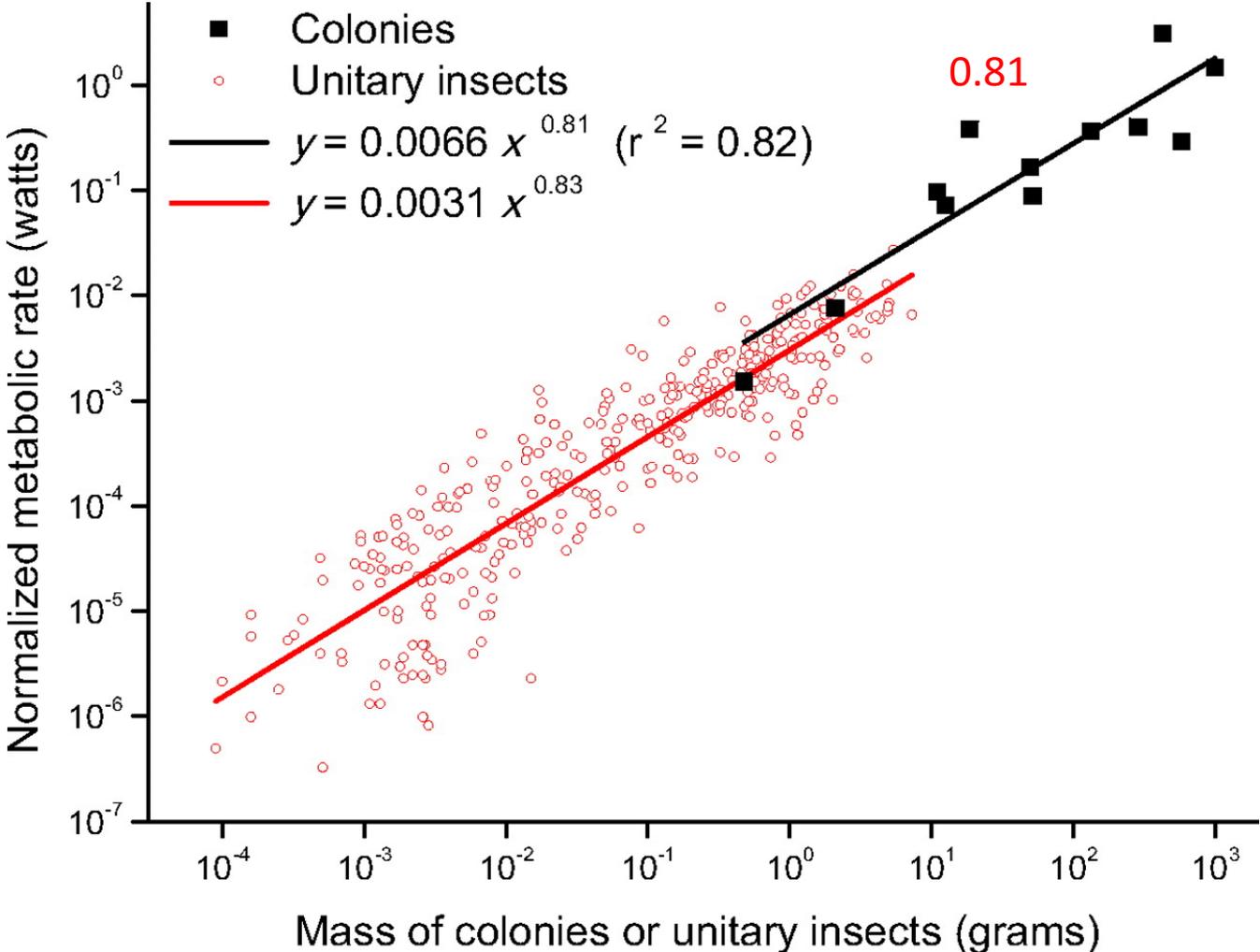
15% of terrestrial animal
biomass

~20,000,000 Ants

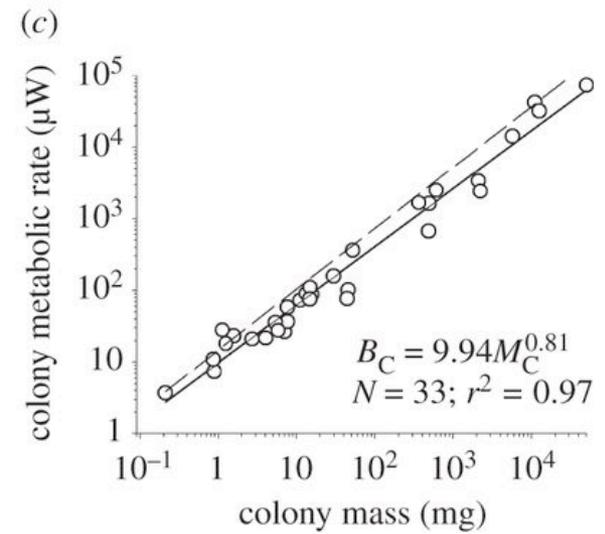
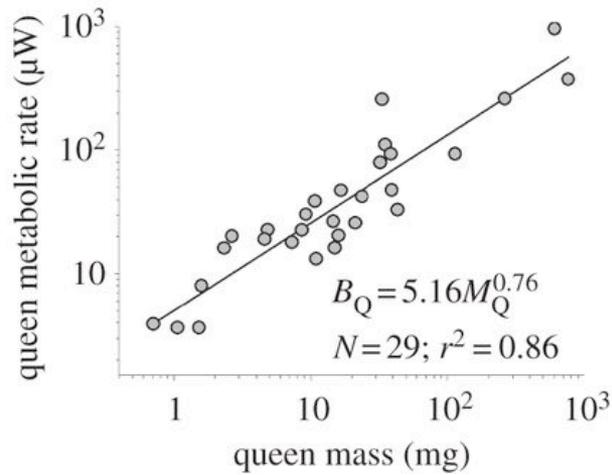
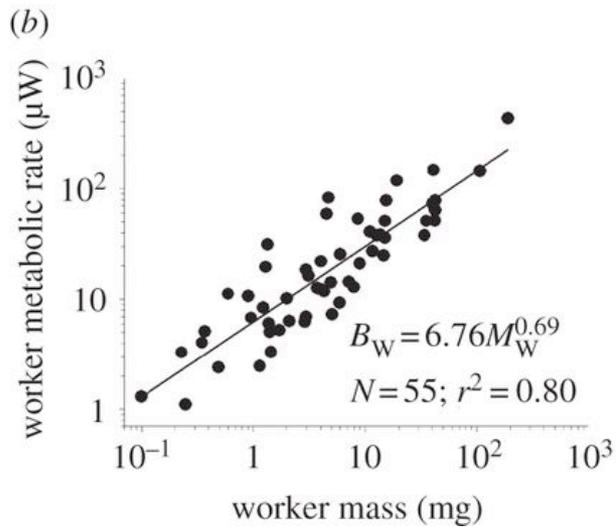
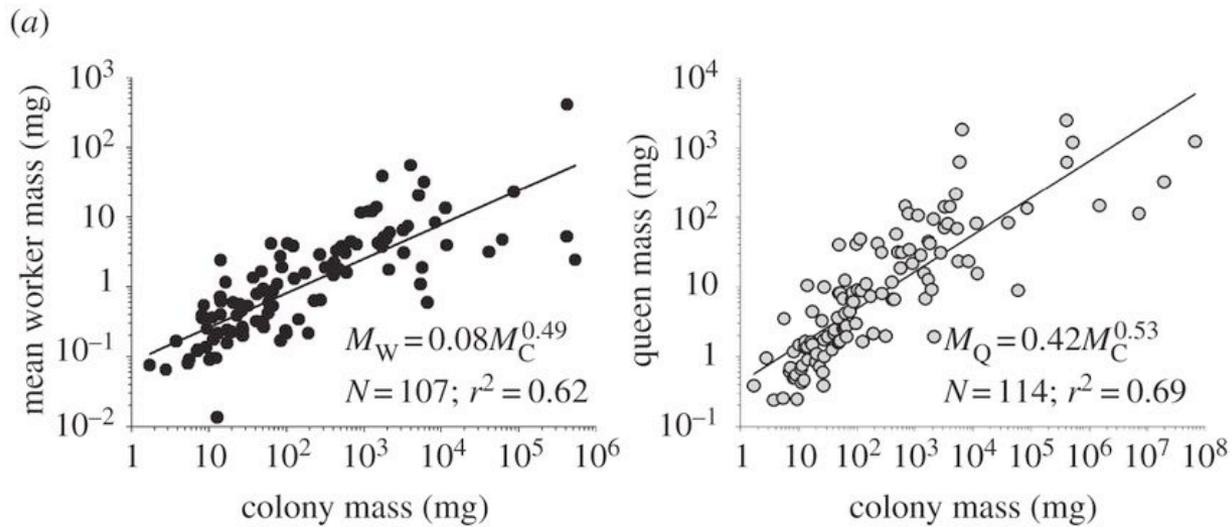


Foraging strategies
adapt to a variety of environments
from simple behaviors
with no central control

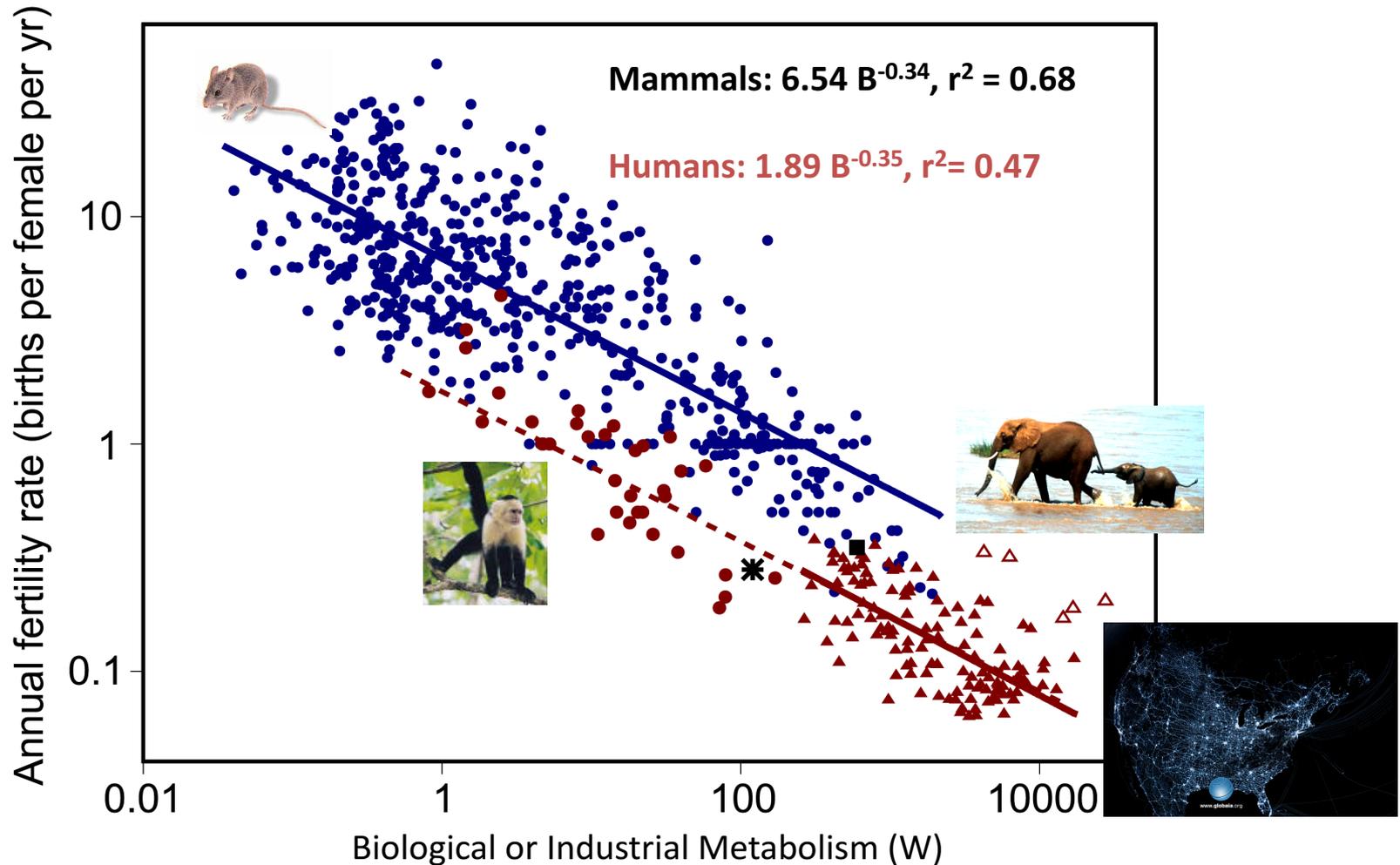
Metabolic rate and body mass for resting unitary insects and whole colonies.

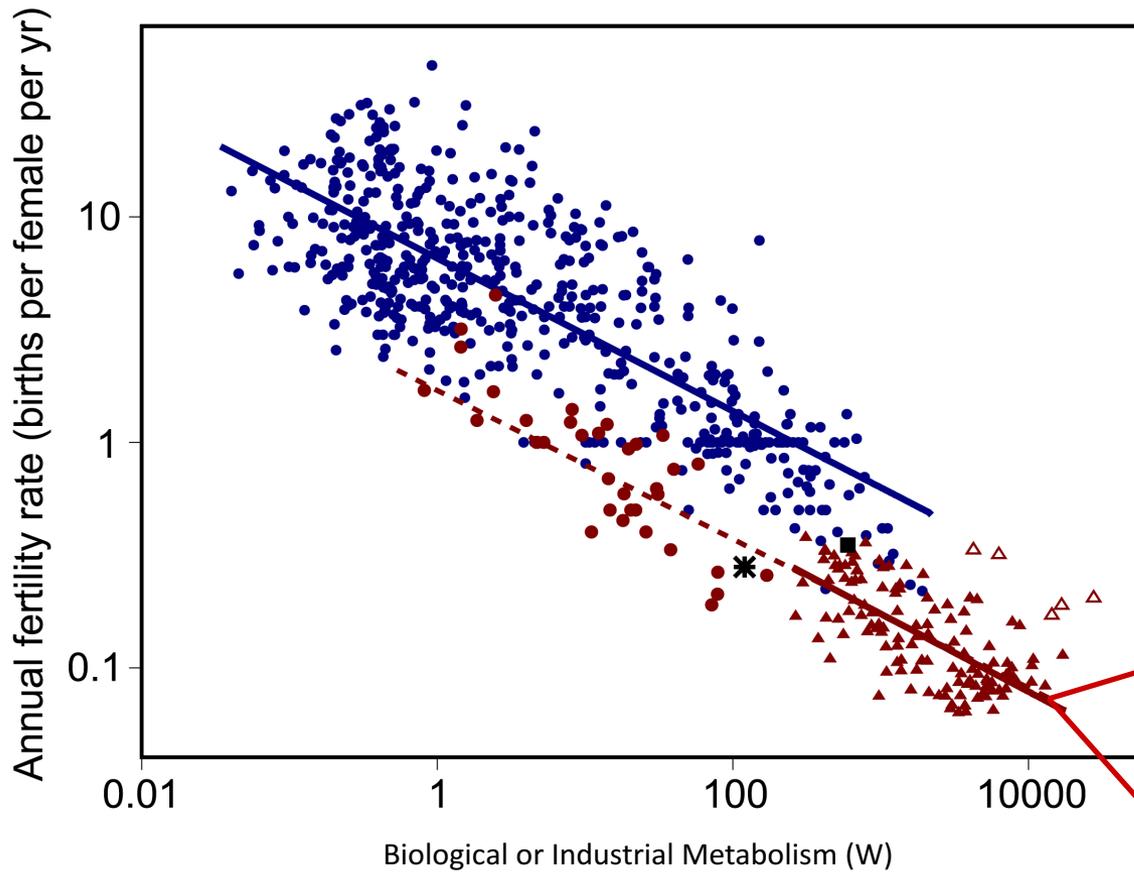


Hou C et al. PNAS 2010;107:3634-3638



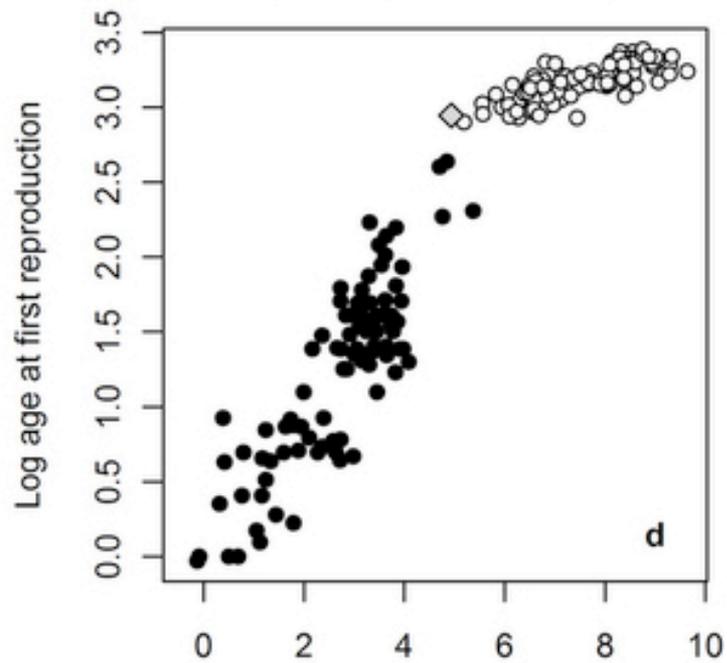
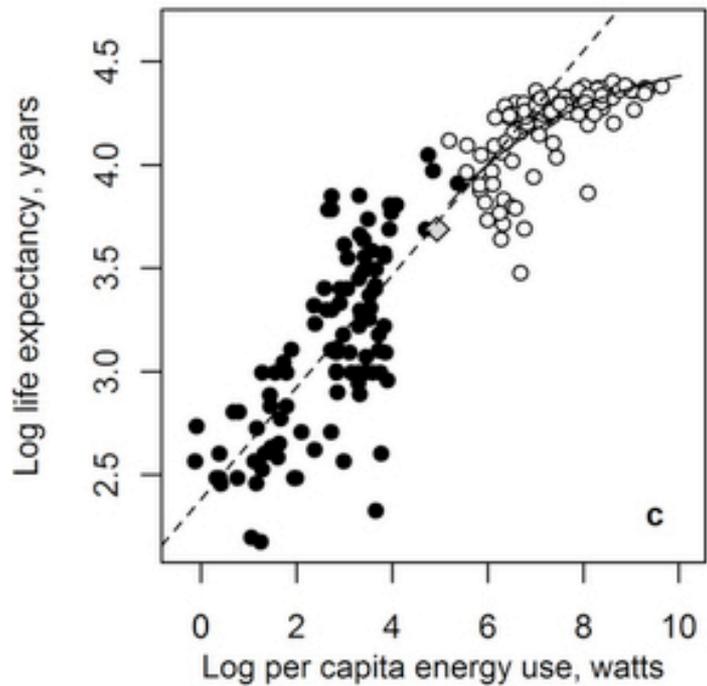
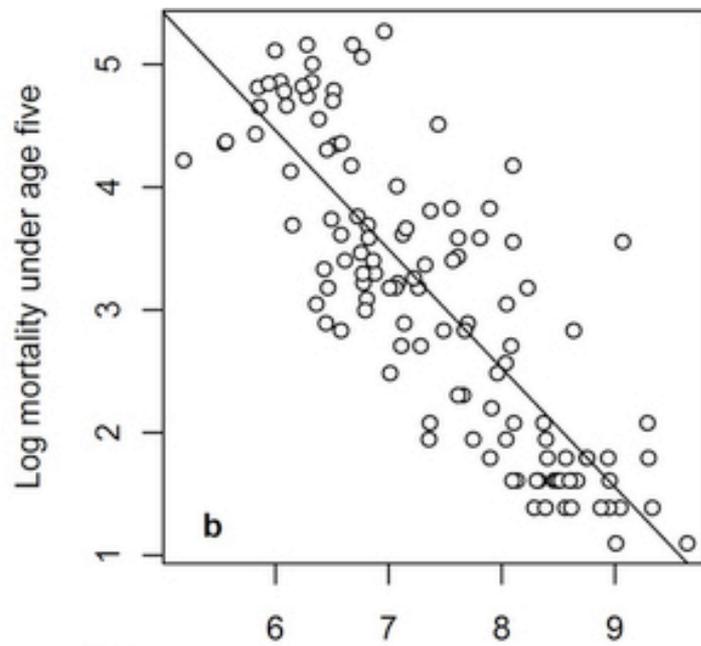
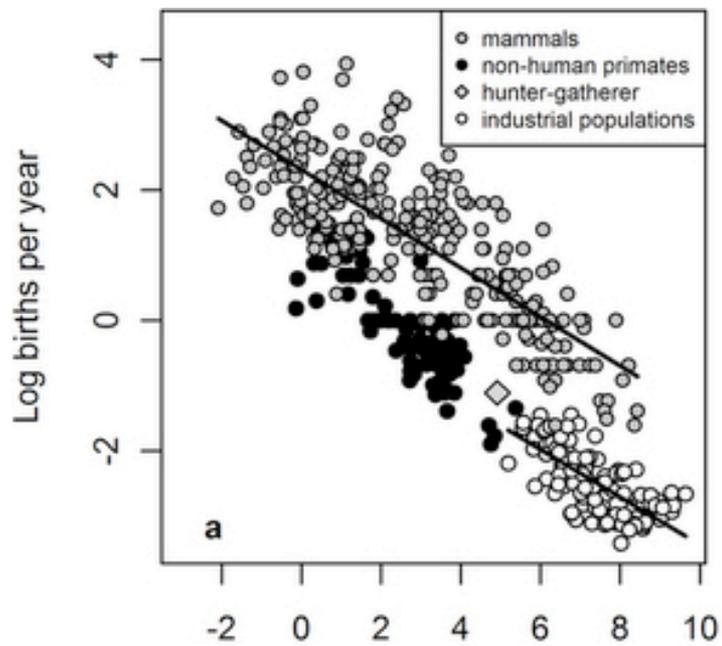
Reproductive Rate vs. Metabolism: Humans and other mammals





Each North American consumes the energy of a 30,000 kg primate

Reproductive rates have dropped accordingly



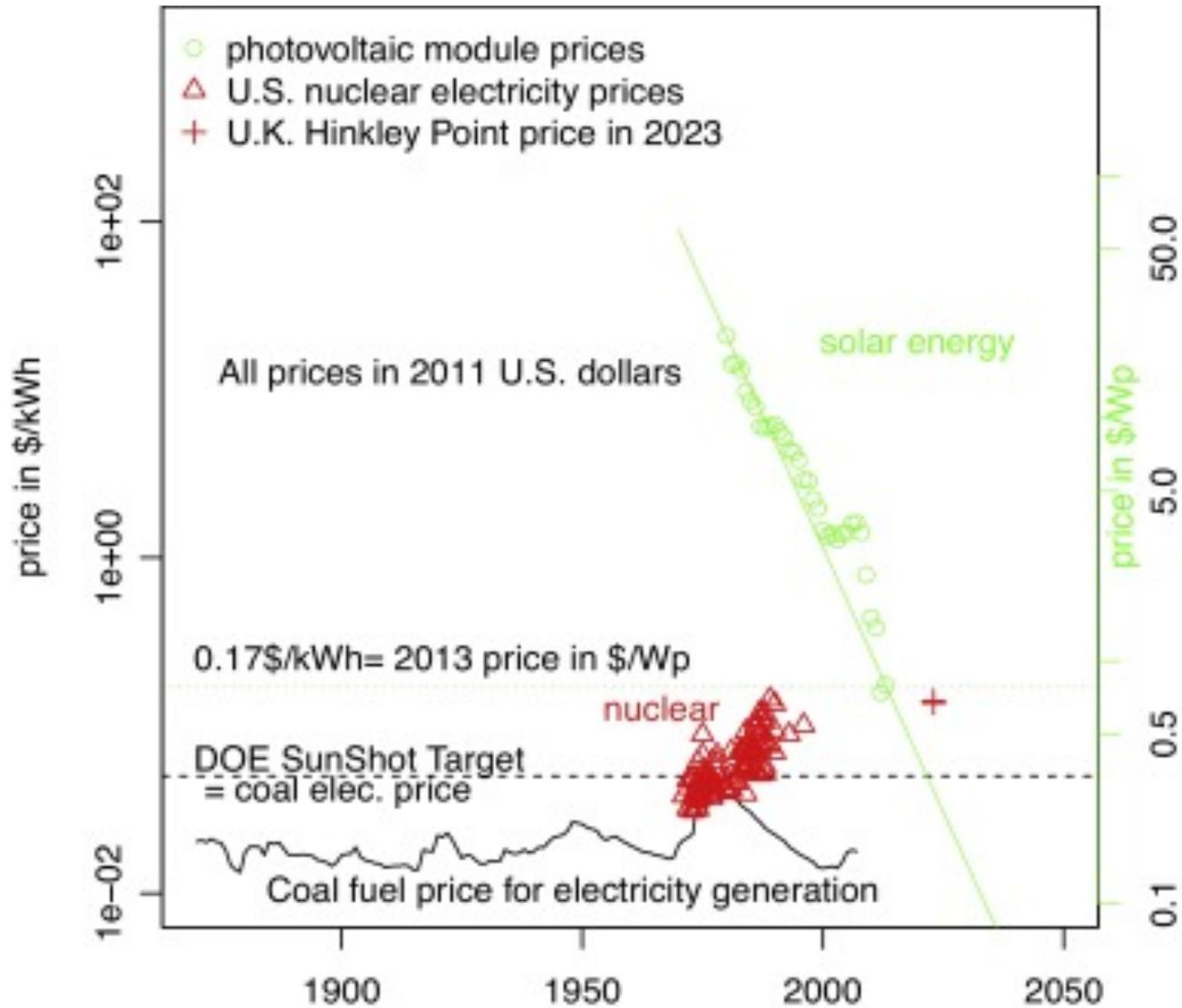


Fig. 1. A comparison of long-term price trends for coal, nuclear power and solar photovoltaic modules. Prices for coal and nuclear power are costs in the US in dollars per kilowatt hour (scale on the left) whereas solar modules are in dollars per watt-peak, i....

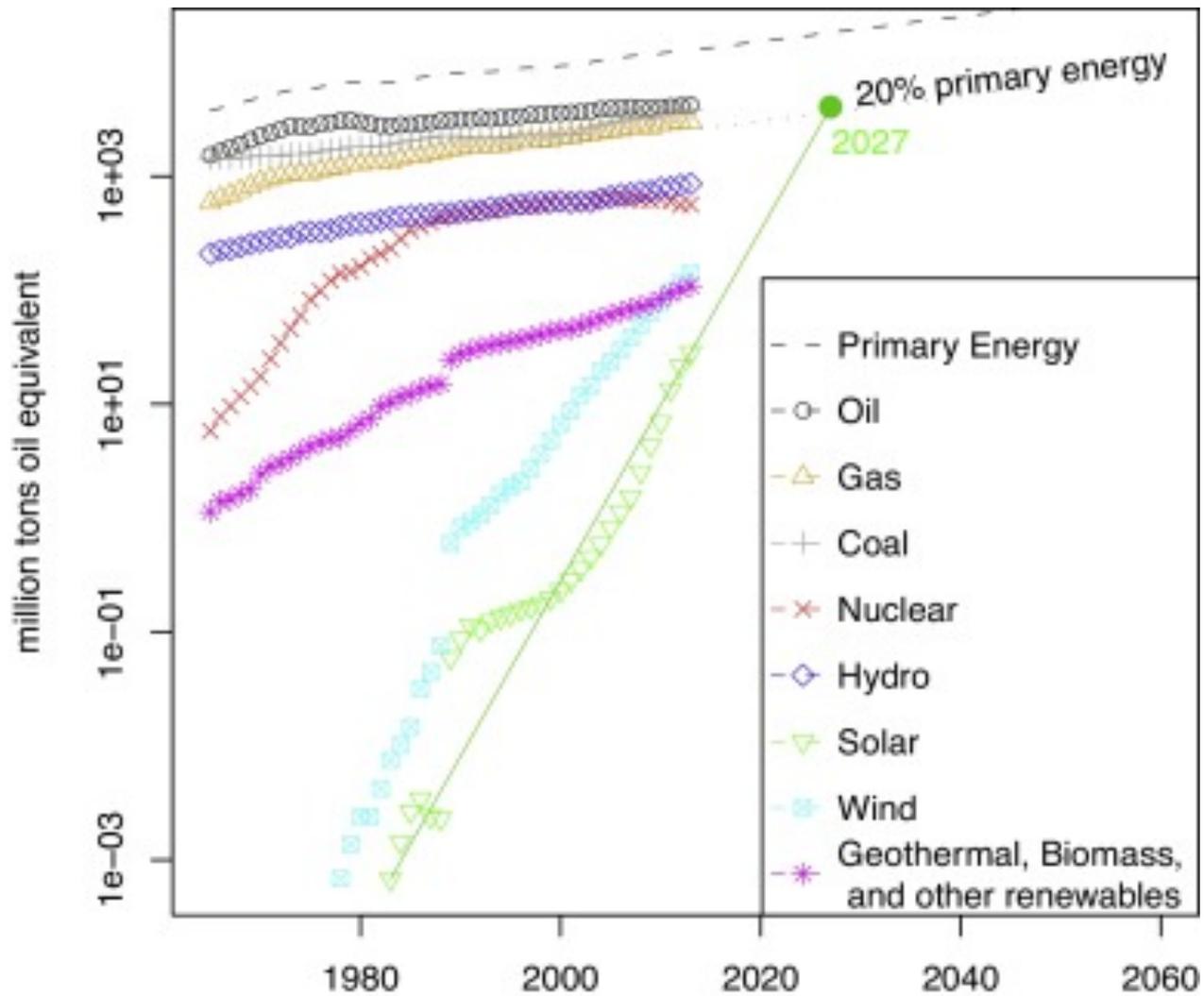


Fig. 20. Global energy consumption due to each of the major sources from BP Statistical Review of World Energy (BP, 2014). Under a projection for solar energy obtained by fitting to the historical data the target of 20% of global primary energy is achieved in ...

A world map where the landmasses are dark, and the cities and urban areas are highlighted with a bright, glowing yellow and white light. The glow is most intense in North America, Europe, and East Asia, with smaller, less intense glows in South America, Africa, and Australia. The oceans are dark, and the poles are white, suggesting ice or snow. The overall effect is a global network of light points.

And why is it everywhere?

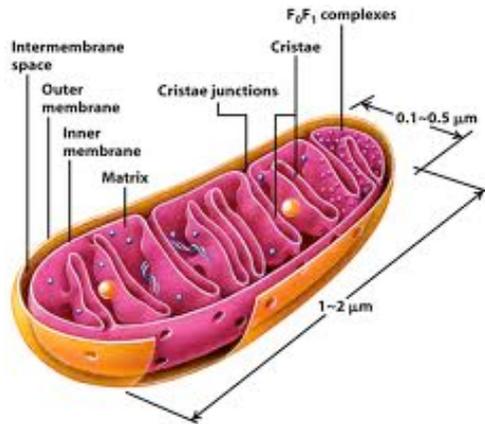
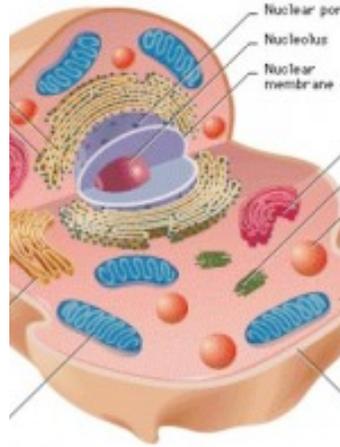
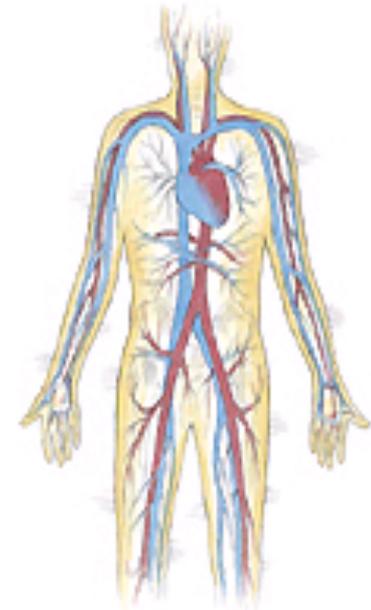


Figure 12-16
Molecular Cell Biology, Sixth Edition
© 2006 W. H. Freeman and Company

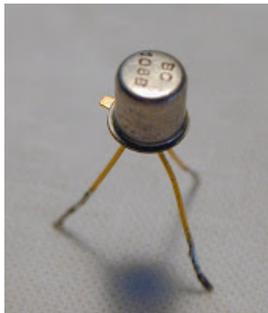
Mitochondrion



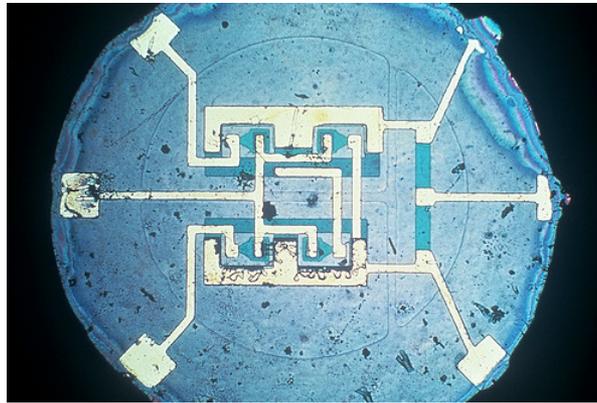
1 to 1000s of
Mitochondria per cell



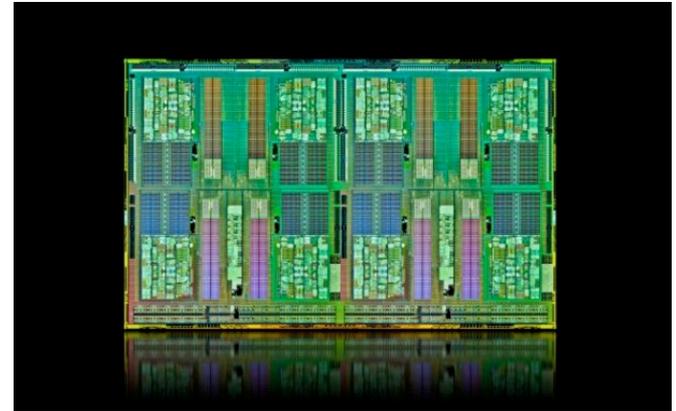
Trillions of mitochondria



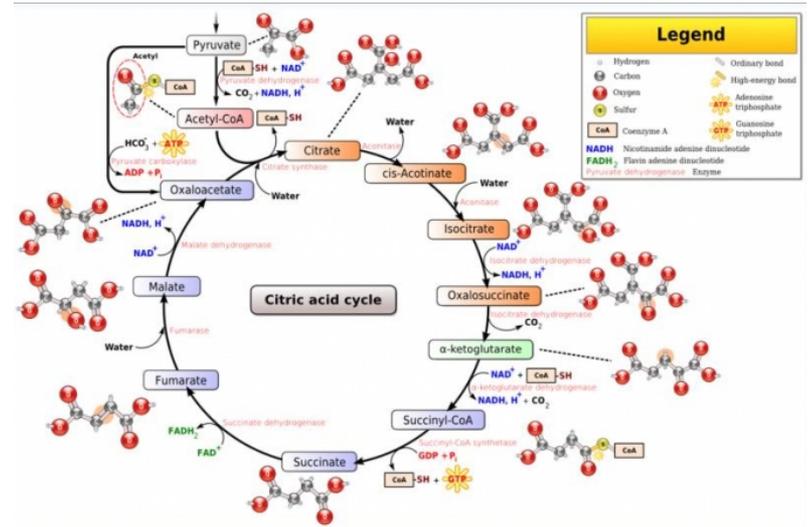
Transistor



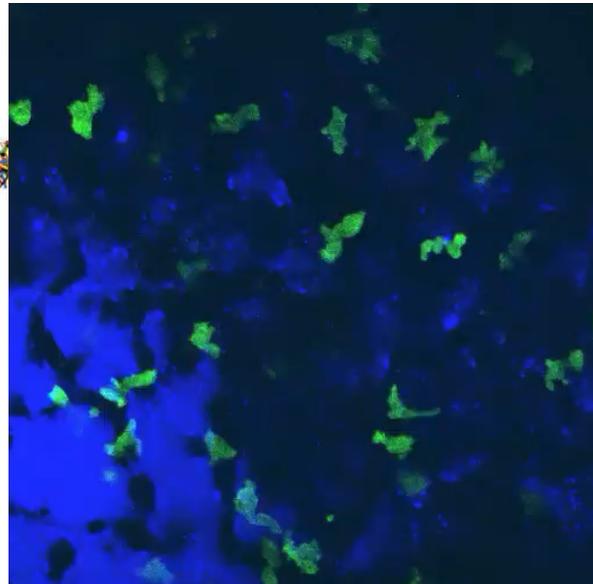
Integrated circuit



AMD Opteron multi-core chip
billions of transistors



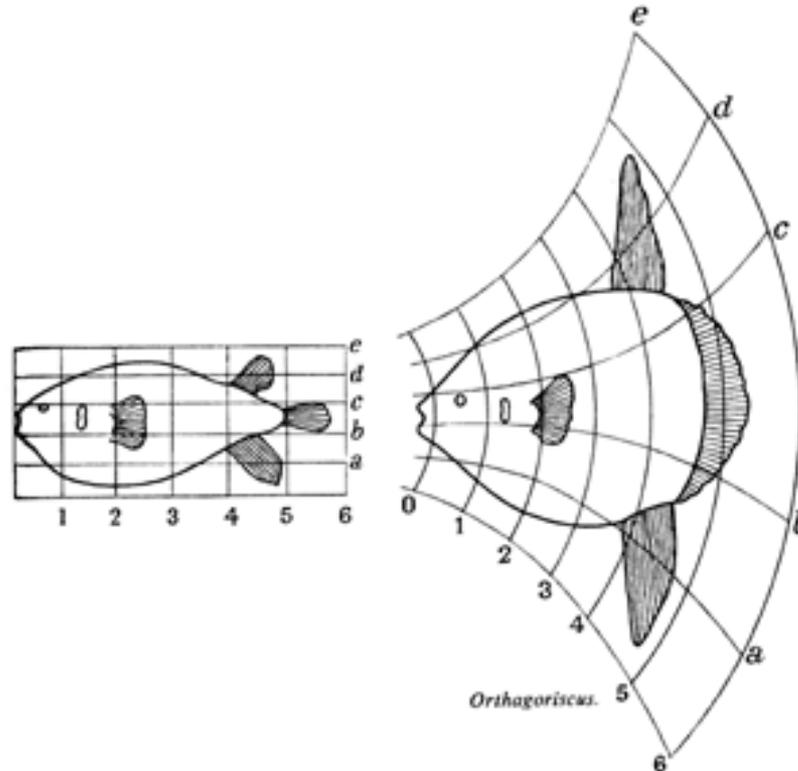
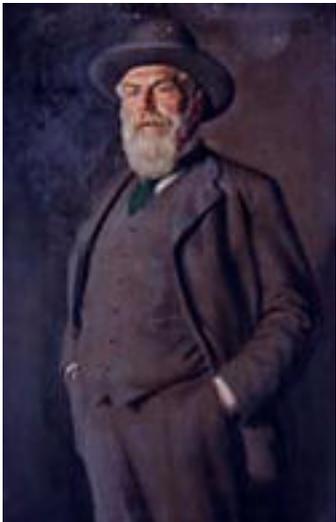
Living systems acquire and transform
energy and information



“We must envisage a living organism as a special kind of system to which **the general laws of physics and chemistry apply.**

And because of the prevalence of **homologies of organization,** we may well suppose, as D’Arcy Thompson has done, that **certain physical processes are of very general occurrence...**”

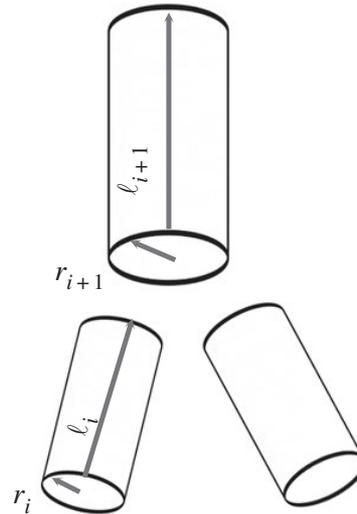
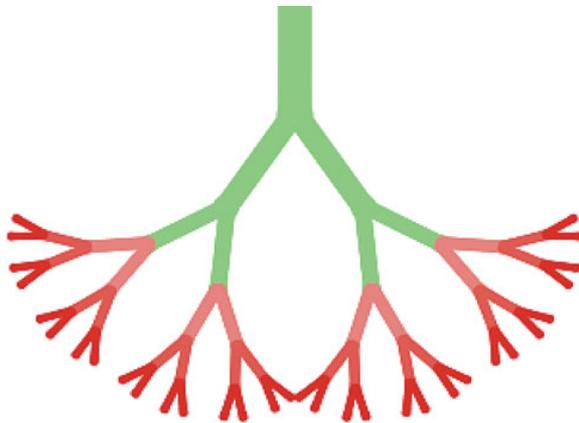
attributed to Alan Turing by Evelyn Fox Keller in Making Sense of Life



Energy and time determine scaling in biological and computer designs

Melanie Moses^{1,2,3}, George Bezerra¹, Benjamin Edwards¹, James Brown^{2,3}
and Stephanie Forrest^{1,2,3}

Energy (oxygen) transported through cardiovascular network to mitochondria



Bits delivered by “interconnect” on microprocessors to transistors

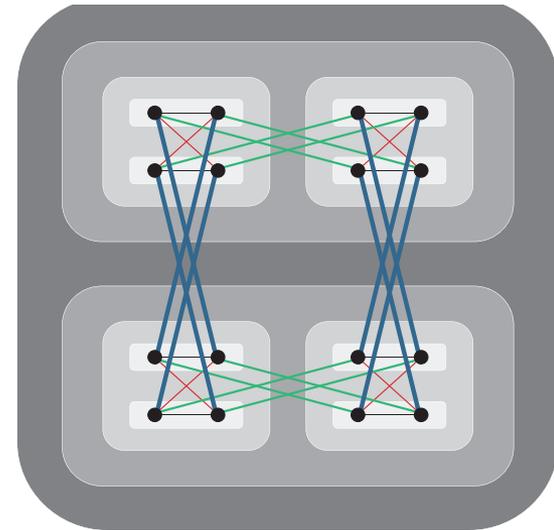


Figure 1. Idealized branching models in biology (a) and computers (c). (a) A cardiovascular tree with branching factor $\lambda = 2$, $H = 5$ hierarchical branchings and $N = 32$ terminal branches at level 0 that represent capillaries. (b) The radius and length of successive branches: D_r defines the relative radius and D_l defines the relative length of pipe or wire between successive hierarchical levels (i and $i + 1$) in both biology (a) and computers (c). (c) The semi-hierarchical branching of logic wires on a computer chip. Each module within a hierarchical level is shaded the same colour. The purple, red, green and blue (thinnest to thickest) wires cross 0, 1, 2 and 3 modules, respectively. The wire lengths and widths increase as they cross more levels according to D_l and D_r . D_w defines the number of wires, determined by the ratio of internal (intra-module) communication per node to external (inter-module) communication per node. Here $D_w = 2$ so that a node is connected to all nodes within a module (in this case only 1) by a purple wire, 1/2 of the nodes in the next hierarchical level by red wires, 1/4 of the nodes in the next level by green wires, and 1/8 of the nodes in the next level by blue wires.

Networks & Diminishing Returns

REPORTS

tracheae, and osmotic and vapor pressure in the plant vascular system). In spite of these differences, these networks exhibit essentially the same scaling laws.

For convenience we shall use the language of the cardiovascular system, namely, aorta, arteries, arterioles, and capillaries; the correspondence to other systems is straightforward. In the general case, the network is composed of N branchings from the aorta (level 0) to the capillaries (level N , denoted here by a subscript c) (Fig. 1C). A typical branch at some intermediate level k has length l_k , radius r_k , and pressure drop Δp_k (Fig. 1D). The volume rate of flow is $\dot{Q}_k = \pi r_k^2 \bar{u}_k$ where \bar{u}_k is the flow velocity averaged over the cross section and, if necessary, over time. Each tube branches into n_k smaller ones (12), so the total number of branches at level k is $N_k = n_0 n_1 \dots n_k$. Because fluid is conserved as it flows through the system

$$\dot{Q}_0 = N_0 \dot{Q}_k = N_k \pi r_k^2 \bar{u}_k = N_c \pi r_c^2 \bar{u}_c \quad (2)$$

which holds for any level k . We next introduce the important assumption, the second above, that the terminal units (capillaries) are invariant, so r_c, l_c, \bar{u}_c , and, consequently, Δp_c are independent of body size. Because the fluid transports oxygen and nutrients for metabolism, $\dot{Q}_0 \propto B$; thus, if $B \propto M^a$ (where a will later be determined to be 3/4), then $\dot{Q}_0 \propto M^a$. Equation 2 therefore predicts that the total number of capillaries must scale as B , that is, $N_c \propto M^a$.

To characterize the branching, we introduce scale factors $\beta_k \equiv r_{k+1}/r_k$ and $\gamma_k \equiv l_{k+1}/l_k$. We shall prove that in order to minimize the energy dissipated in the system in the sense of the third principle above, the network must be a conventional self-similar fractal in that $\beta_k = \beta$, $\gamma_k = \gamma$, and $n_k = n$, all independent of k (an important exception is β_k in pulsatile systems). For a self-similar fractal, the number of branches increases in geometric proportion ($N_k = n^k$) as their size geometrically decreases from level 0 to level N . Before proving self-similarity, we first examine some of its consequences.

Because $N_c = n^N$, the number of generations of branches scales only logarithmically with size

$$N = \frac{a \ln(M/M_0)}{\ln n} \quad (3)$$

where M_0 is a normalization scale for M (13). Thus, a whale is 10^7 times heavier than a mouse but has only about 70% more branchings from aorta to capillary. The total volume of fluid in the network ("blood" volume V_b) is

$$V_b = \sum_{k=0}^N N_k V_k = \sum_{k=0}^N \pi r_k^2 l_k n^k$$

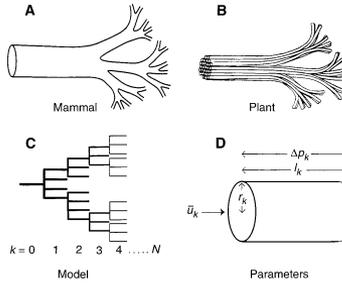


Fig. 1. Diagrammatic examples of segments of biological distribution networks: (A) mammalian circulatory and respiratory systems composed of branching tubes; (B) plant vessel-bundle vascular system composed of diverging vessel elements; (C) topological representation of such networks, where k specifies the order of the level, beginning with the aorta ($k = 0$) and ending with the capillary ($k = N$); and (D) parameters of a typical tube at the k th level.

$$= \frac{(n\gamma\beta^2)^{-(N+1)} - 1}{(n\gamma\beta^2)^{-1} - 1} n^N V_c \quad (4)$$

where the last expression reflects the fractal nature of the system. As shown below, one can also prove from the energy minimization principle that $V_b \propto M$. Because $n\gamma\beta^2 < 1$ and $N \gg 1$, a good approximation to Eq. 4 is $V_b = V_c / (1 - n\gamma\beta^2) = V_c (\gamma\beta^2)^{-N} / (1 - n\gamma\beta^2)$. From our assumption that capillaries are invariant units, it therefore follows that $(\gamma\beta^2)^{-N} \propto M$. Using this relation in Eq. 3 then gives

$$a = - \frac{\ln n}{\ln(\gamma\beta^2)} \quad (5)$$

To make further progress requires knowledge of γ and β . We shall show how the former follows from the space-filling fractal requirement, and the latter, from the energy minimization principle.

A space-filling fractal is a natural structure for ensuring that all cells are serviced by capillaries. The network must branch so that a group of cells, referred to here as a "service volume," is supplied by each capillary. Because $r_c \ll l_c$ and the total number of branchings N is large, the volume supplied by the total network can be approximated by the sum of spheres whose diameters are that of a typical k th-level vessel, namely $4/3\pi(l_k/2)^3 N_k$. For large N , this estimate does not depend significantly on the specific level, although it is most accurate for large k . This condition, that the fractal be volume-preserving from one generation to the next, can therefore be expressed as $4/3\pi(l_k/2)^3 N_k \approx 4/3\pi(l_{k+1}/2)^3 N_{k+1}$. This relation gives $\gamma^3 \beta^3 \approx (l_{k+1}/l_k)^3 \approx N_k/N_{k+1} = 1/n$, showing that $\gamma \approx n^{1/3} \approx \gamma$ must be independent of k . This result for γ_k is a general property of all space-filling fractal systems that we consider.

The 3/4 power law arises in the simple case of the classic rigid-pipe model, where the branching is assumed to be area-preserving,

that is, the sum of the cross-sectional areas of the daughter branches equals that of the parent, so $\pi r_k^2 = n\pi r_{k+1}^2$. Thus, $\beta_k \equiv r_{k+1}/r_k = n^{-1/2} = \beta$, independent of k . When the area-preserving branching relation, $\beta = n^{-1/2}$, is combined with the space-filling result for γ , Eq. 5 yields $a = 3/4$, so $B \propto M^{3/4}$. Many other scaling laws follow. For example, for the aorta, $r_0 = \beta^{-N} r_c = N^{1/2} r_c$ and $l_0 = \gamma^{-N} l_c = N^{1/3} l_c$, yielding $r_0 \propto M^{3/8}$ and $l_0 \propto M^{1/4}$. This derivation of the $a = 3/4$ law is essentially a geometric one, strictly applying only to systems that exhibit area-preserving branching. This property has the further consequence, which follows from Eq. 2, that the fluid velocity must remain constant throughout the network and be independent of size. These features are a natural consequence of the idealized vessel-bundle structure of plant vascular systems (Fig. 1B), in which area-preserving branching arises automatically because each branch is assumed to be a bundle of n^{N-k} elementary vessels of the same radius (11). Pulsatile mammalian vascular systems, on the other hand, do not conform to this structure, so for them, we must look elsewhere for the origin of quarter-power scaling laws.

Some features of the simple pipe model remain valid for all networks: (i) The quantities γ and β play a dual scaling role: they determine not only how quantities scale from level 0 (aorta) to N (capillary) within a single organism of fixed size, but also how a given quantity scales when organisms of different masses are compared. (ii) The fractal nature of the entire system as expressed, for example, in the summation in Eq. 4 leads to a scaling different from that for a single tube, given by an individual term in the series. These network systems must therefore be treated as a complete integrated unit; they cannot realistically be modeled by a single or a few representative vessels. (iii) The scaling with M does not

Fractal Networks Generate 3/4 powers

Centralized hierarchical, fractal branching

1. Constant branching ratio,

2. Area preserving

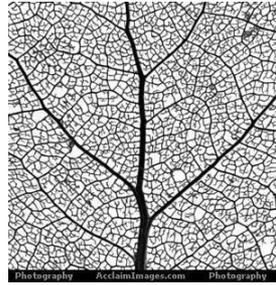
3. Space filling

4. Invariant terminal units

-Capillaries same length, radius & delivery capacity

-Metabolism proportional to # of capillaries

5. Network volume proportional to mass



Fractal Networks Generate 3/4 powers

Centralized hierarchical, fractal branching

1. Constant branching ratio, b

2. Area preserving $N_k A_k = c$

3. Space filling $\frac{l_{k+1}}{l_k} = b^{1/3}$

4. Invariant terminal units

-Capillaries same length, radius & delivery capacity

-Metabolism proportional to # of capillaries

5. Network volume proportional to mass



Metabolic Rate is proportional to the number of capillaries

To double metabolic rate, double the number of capillaries



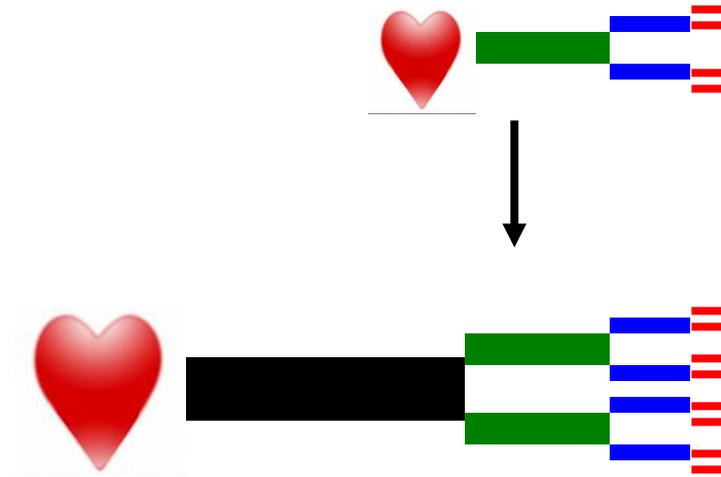
Additional network (black)
is needed to connect the 2 smaller networks

$$V_{net} = \pi b^k A_{cap} l_{cap} \sum_{i=0}^k b^{i/3}$$

$$V_{net} \propto (b^k)^{(4/3)}$$

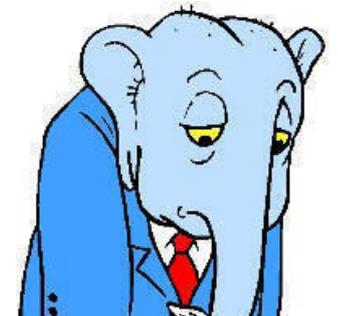
$$V_{net} \propto N_{cap}^{(4/3)} \propto B^{(4/3)}$$

$$B \propto V_{net}^{3/4}$$



Increasing Volume 100 times increases metabolic delivery 30 times

Diminishing returns: Network size grows faster than network delivery rate



Dec Alpha H-tree (1994), a 2D WBE fractal network

A centralized network that delivered a timing signal

Wire lengths and radii follow WBE predictions in 2D

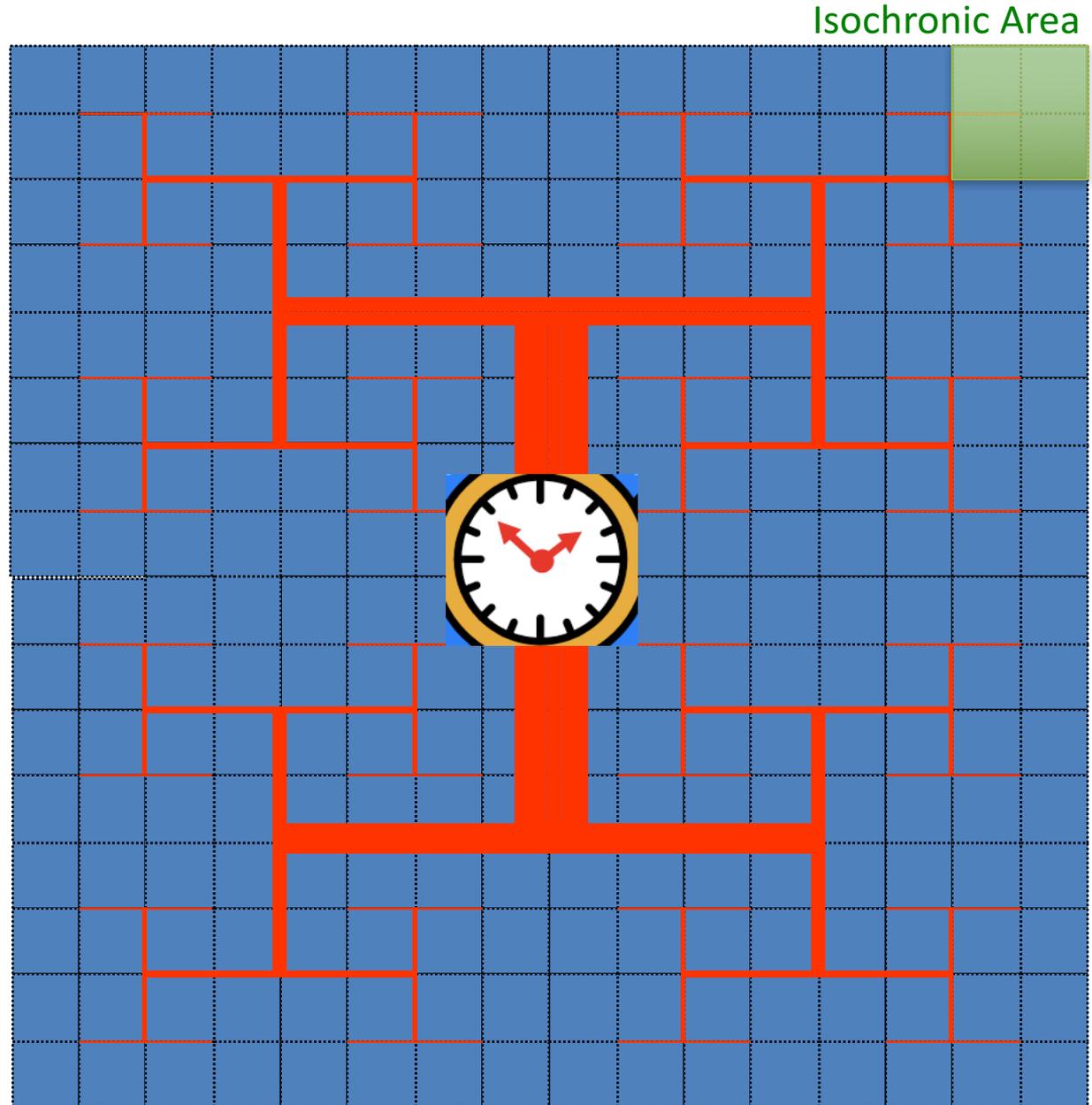
Allowed unprecedented speed (300 MHz)

Clock speed is limited by the isochronic area (last mile)

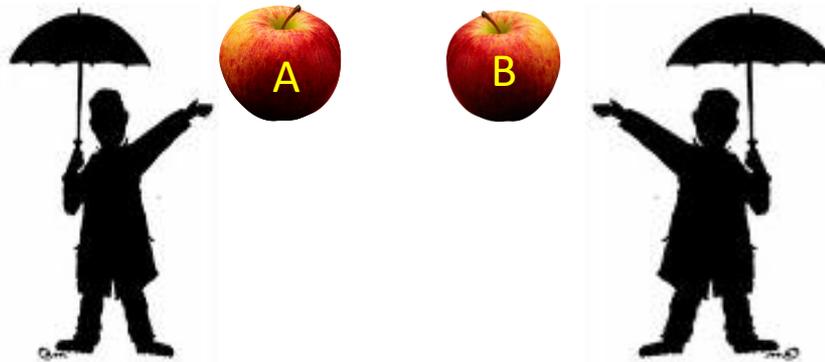
Clock area $\sim A_{\text{chip}}^{3/2}$

The clock consumed 40% of the chip's power

Diminishing Returns



Scaling in Information Networks: Increasing Returns in Information Exchange



If you have an apple and I have an apple and we exchange apples
then **you and I will still each have one apple.**
But if you have an idea and I have an idea and we exchange these ideas,
then each of us will have two ideas.

--George Bernard Shaw



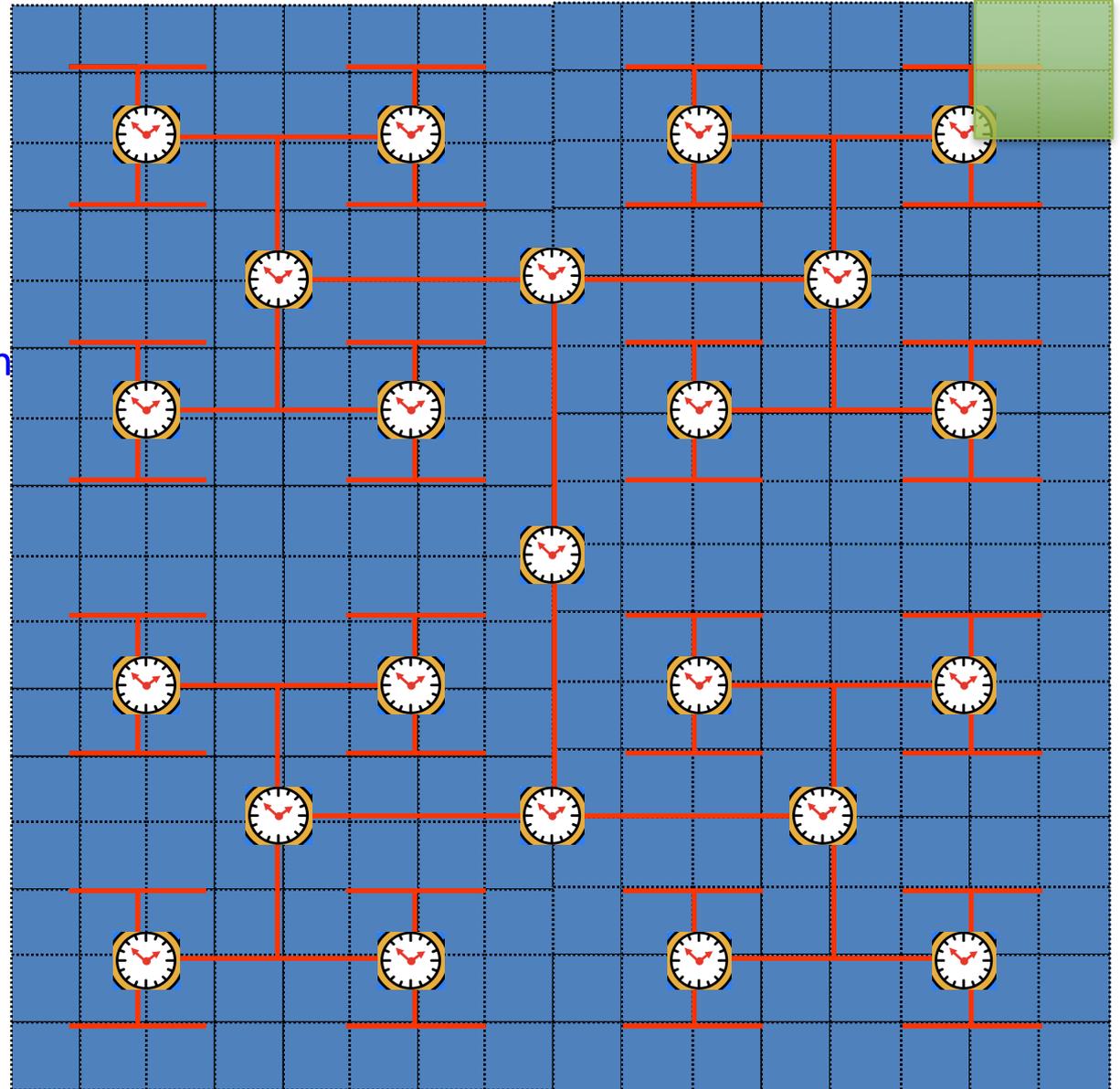
Partially Decentralized Intel Itanium 2 H-tree (2004)

Information can be copied

Amplifiers regenerate
clock signal at each branch

Decentralized communication
generates linear scaling
of clock power & area
with chip area

Synchronize more transistors
with less power

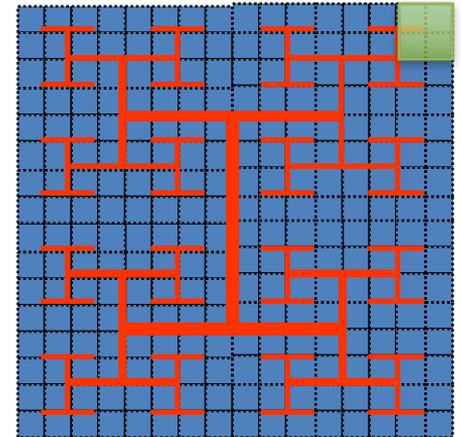


Microprocessors: Fractal Networks, different scaling

Centralized, hierarchical, fractal branching networks
Efficient networks: Max. energy delivery, min. energy dissipation

Assumptions

1. Constant branching ratio b
2. Space filling (becomes 2D area filling) $\frac{l_{k+1}}{l_k} = b^{1/2}$
3. Area preserving branching: **No**—Rent's rule
4. Invariant terminal units: **No**
 - Transistors (service volumes) shrink as network grows
 - Throughput proportional to # of transistors
5. Network volume proportional to mass:
No, Metal layers accommodate extra wire



Assumptions

- Living systems and computer chips are designed to maximize the rate at which resources are delivered to terminal nodes of a network and to minimize the energy dissipated as it is delivered and processed.
 - Minimize Energy dissipation & Delivery Time
(Minimize the energy-time product)
 - Explicitly consider energy & time in the network AND nodes
 - matching supply and demand (pipelining)
- Biology: minimize energy dissipated in the network & maximize metabolic rate
- Computers: minimize total energy consumption on the chip and maximize rate that bits are processed (MIPS)

$$\min(E_{\text{sys}} \times T_{\text{sys}}) = \min_{D_r, D_w, D_1} \left(RN + \frac{N^2}{Q} \right).$$

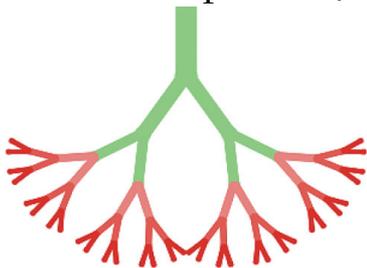
$$R = \sum_{i=0}^H \frac{8\mu l_i}{\pi r_i^4} \frac{1}{n_i} = \frac{8\mu l_0}{\pi r_0^4} \lambda^{-H} \sum_{i=0}^H \lambda^{i(1/D_1 - 4/D_r + 1)},$$

$$Q \propto u_0 N^{2/D_r}$$

$$\min_{D_r} \left(RN + \frac{N^2}{Q} \right) \propto l_0 + u_0^{-1} N^{2-2/D_r}.$$

Energy dissipation in the network is minimized when $D_r = 2$ (area preserving branching)

$$D_r = 24/11 = 2.18.$$



general energy – time
minimization

mammals

E_{net}	$l_0 u_0 N^{2/D_r - 1}$	$N^{1/12}$
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E_{node}	N	N
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T_{net}	$u_0^{-1} N^{1-2/D_r}$	N^0
------------------	------------------------	-------

T_{node}	$u_0^{-1} N^{1-2/D_r}$	N^0
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$E_{\text{sys}} \times T_{\text{sys}}$	$l_0 + u_0^{-1} N^{2-2/D_r}$	$N^{1/12} + N$
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computers

E_{net}	N^{1-1/D_1}	$N^{1/2}$
------------------	---------------	-----------

E_{node}	N^{1-1/D_1}	$N^{1/2}$
-------------------	---------------	-----------

T_{net}	N^0	N^0
------------------	-------	-------

T_{node}	N^{-1/D_1}	$N^{-1/2}$
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$E_{\text{sys}} \times T_{\text{sys}}$	$N^{1-1/D_1} + N^{1-1/D_1}$	$N^{1/2} + N^{1/2}$
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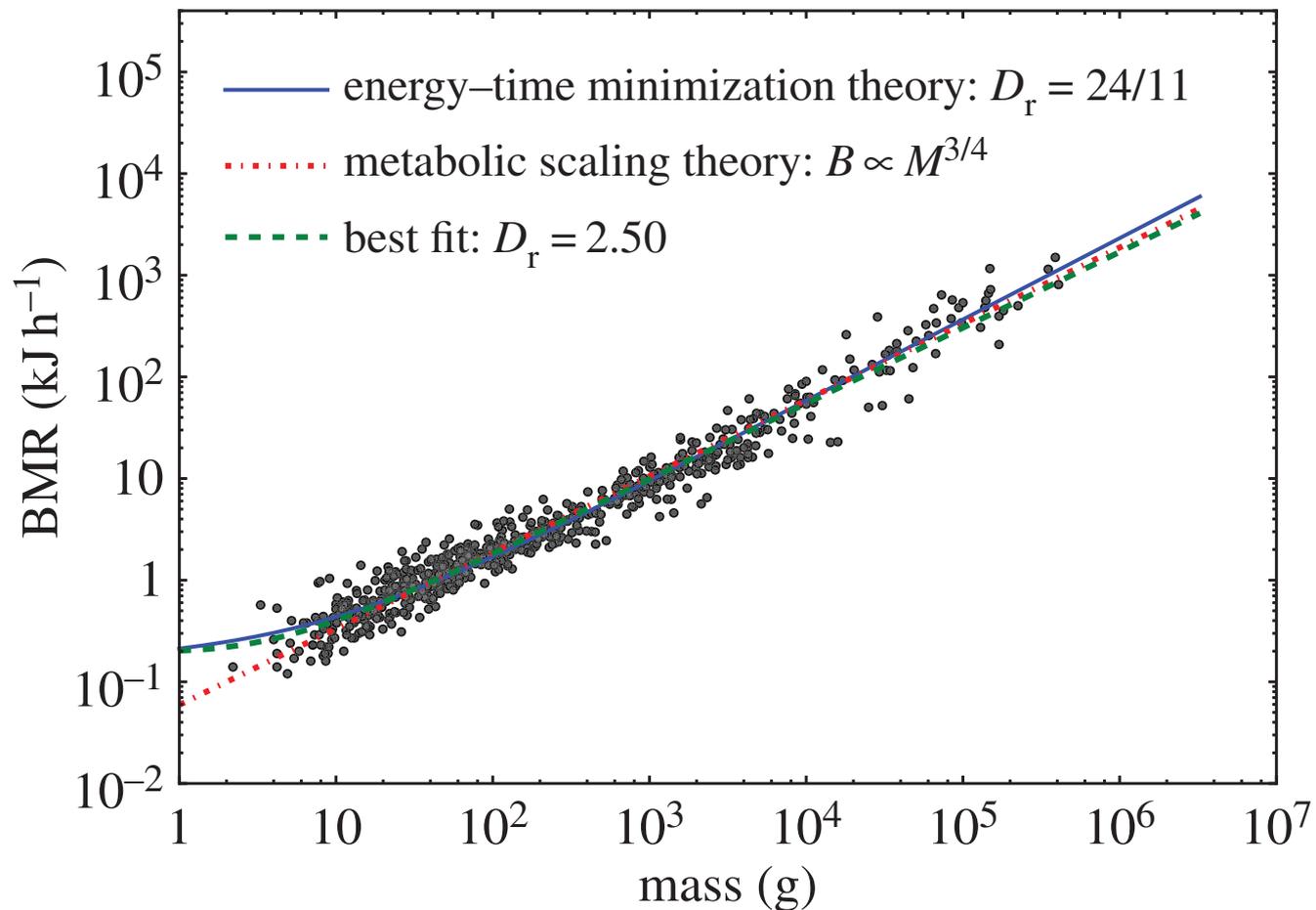
We predict the optimal D_r given that

- blood must slow ($D_r > 2$)
- match the delivery rate of oxygen by the network to the consumption rate in the nodes (both invariant wrt N)

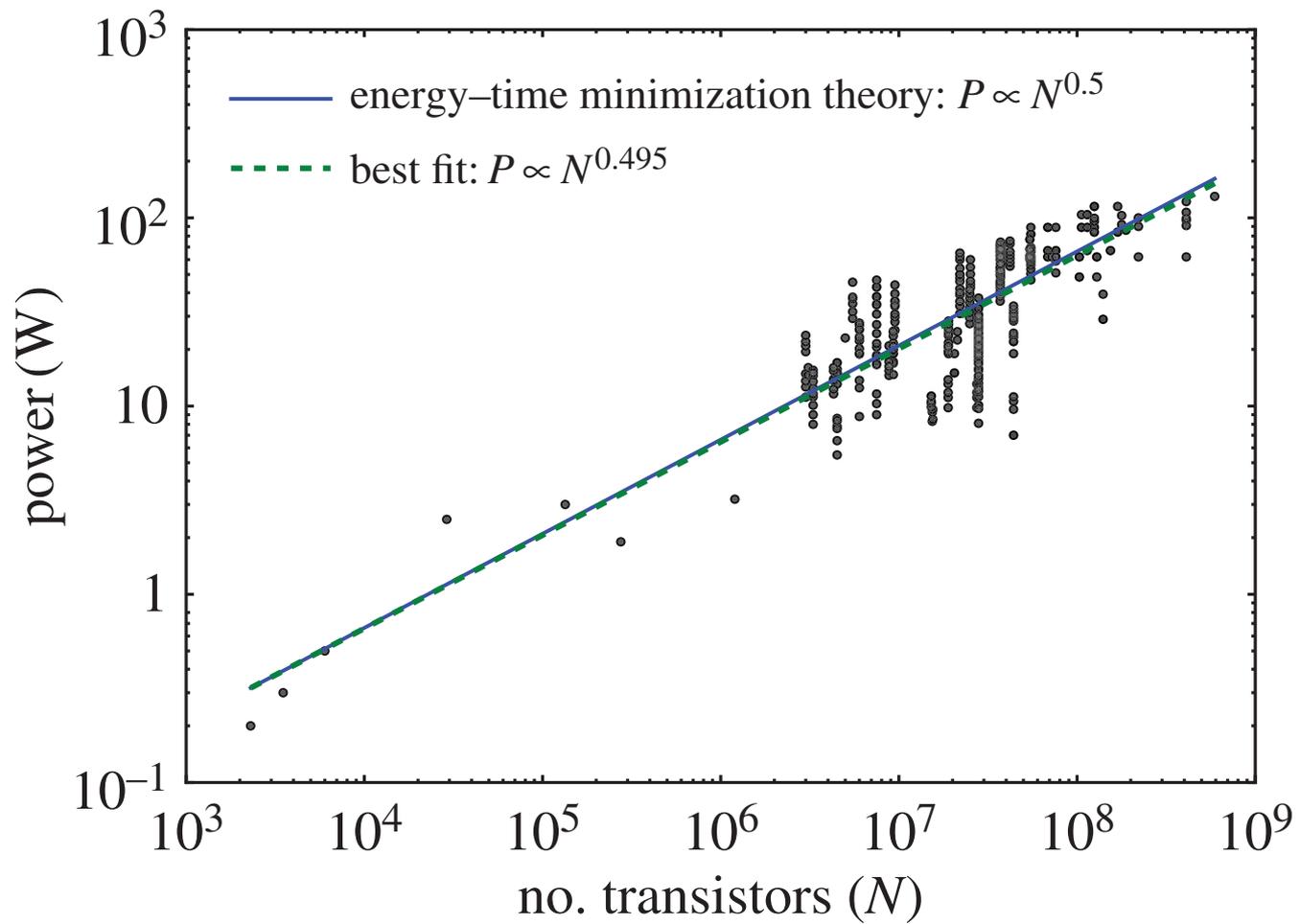
Relax area preserving assumption

Minimize time & energy dissipation: $D_r = 2.18$

Consider energy to drive network + energy processed in nodes

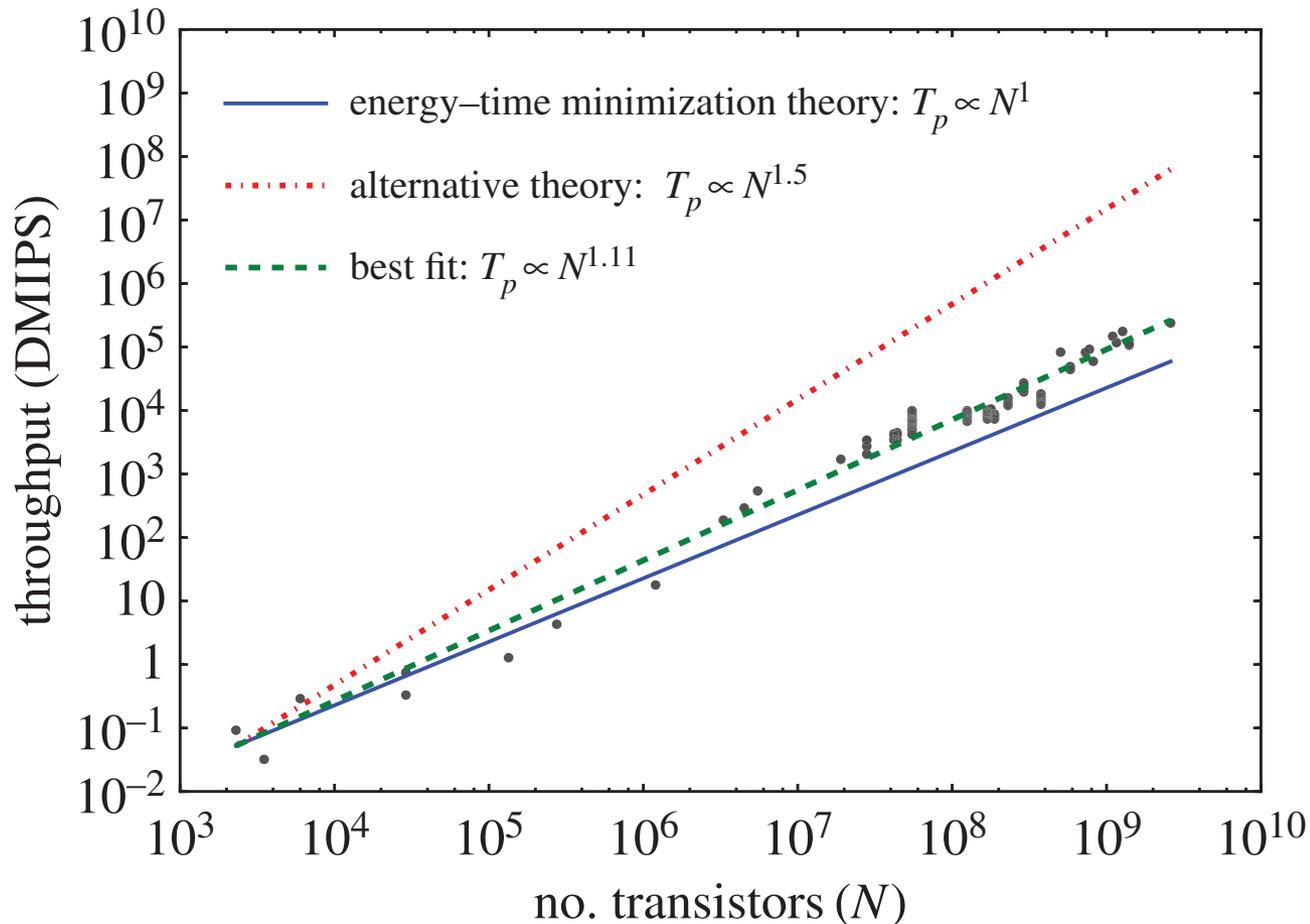


Power vs Size on Chips



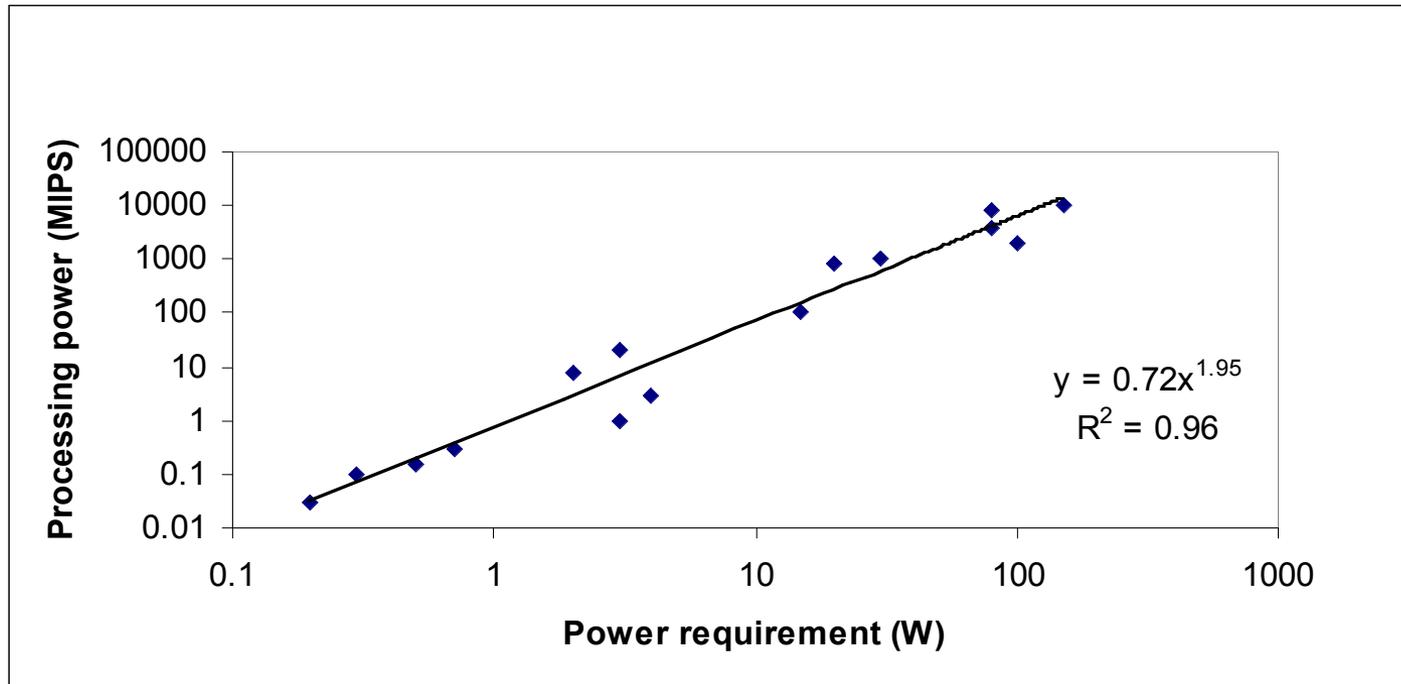
“Hegemony of the network”

Linear scaling between throughput and # of transistors



Power scaling: Increasing returns

Thousand-fold increase in power, Million-fold increase in MIPS



1970

2005

In 1970, 100 Watts powered 15 MIPS. In 2005, 6700 MIPS

Transistors perform computations
Power consumption is dominated by wires

Metabolic Scaling

A striking universal(?) pattern

Metabolic rate scales sub-linearly with mass

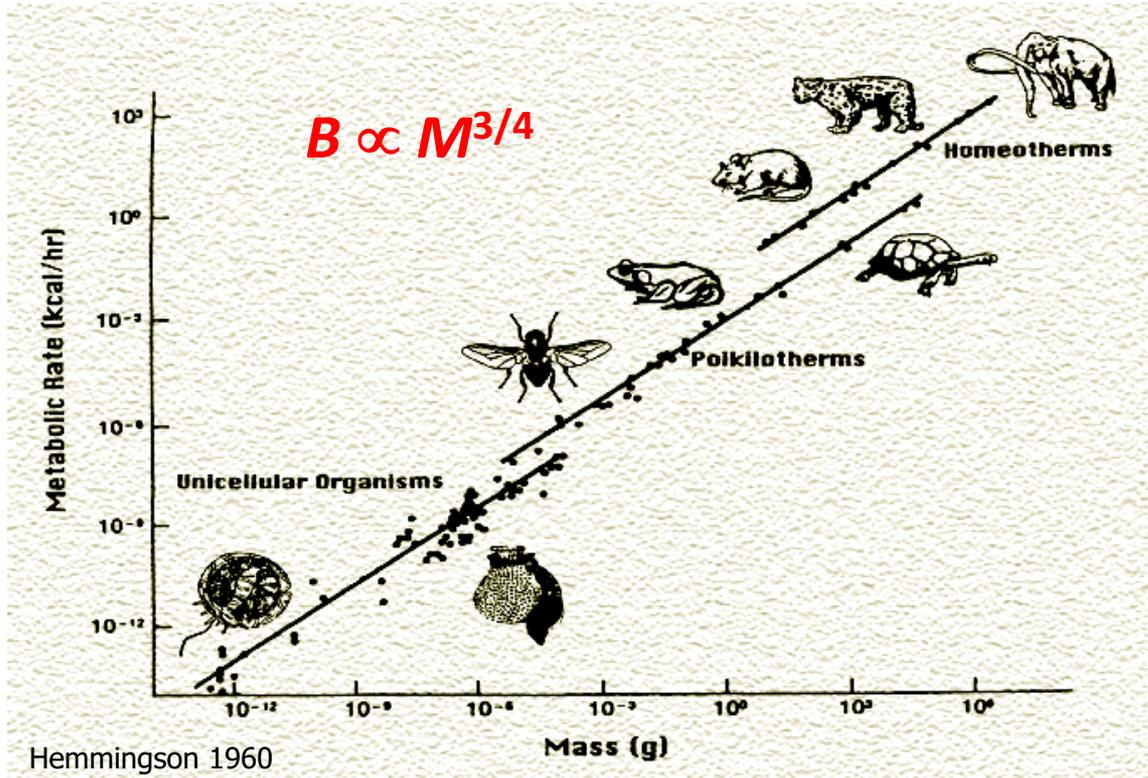
Metabolism is the rate of energy use

Measured as

Oxygen in, or CO₂ out

Food consumption

(for non-growing animals)



Metabolism governs

Physiology

Growth

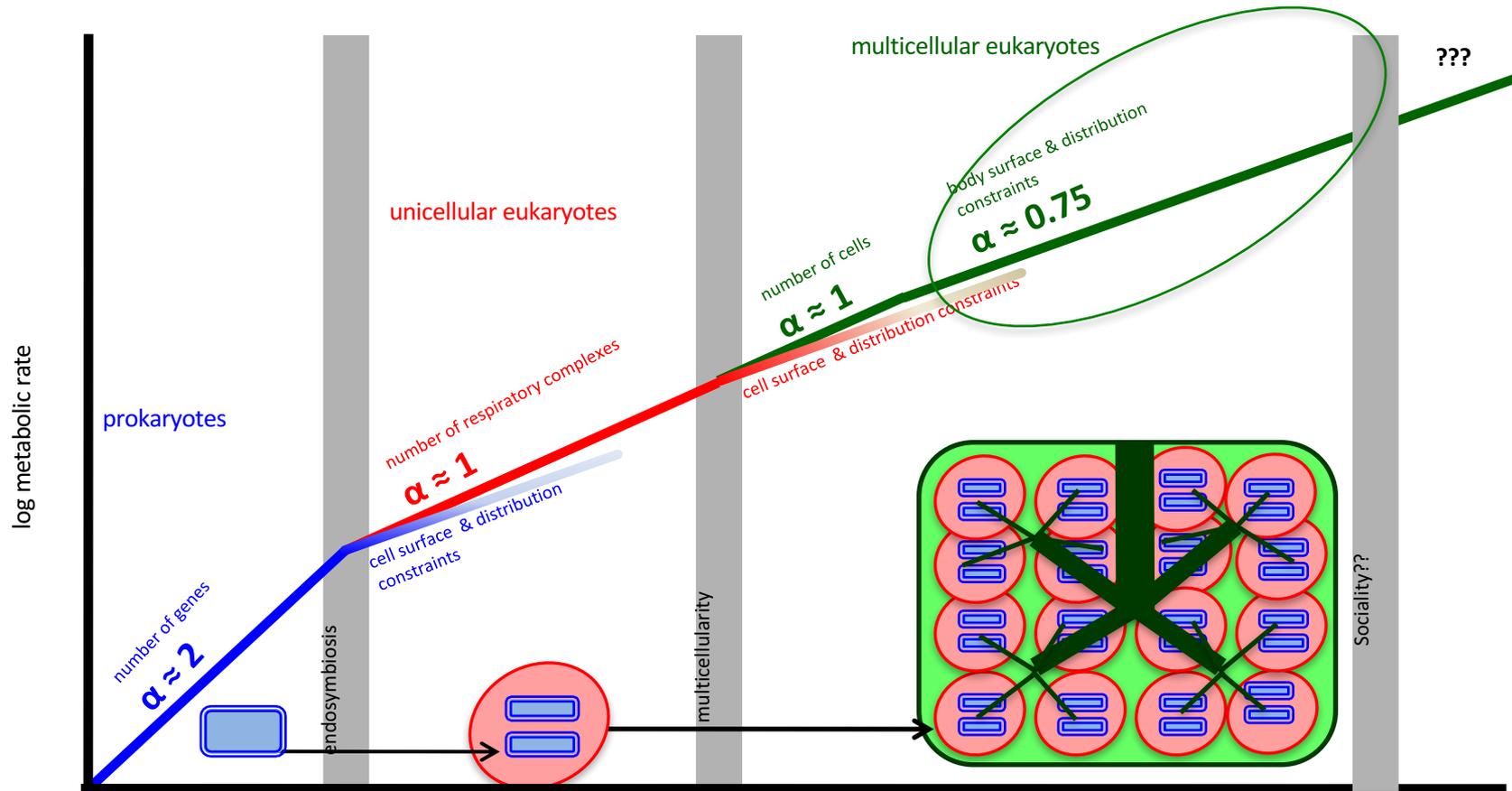
Reproduction

Lifespan

Photosynthesis & carbon flux

Ecosystem dynamics...

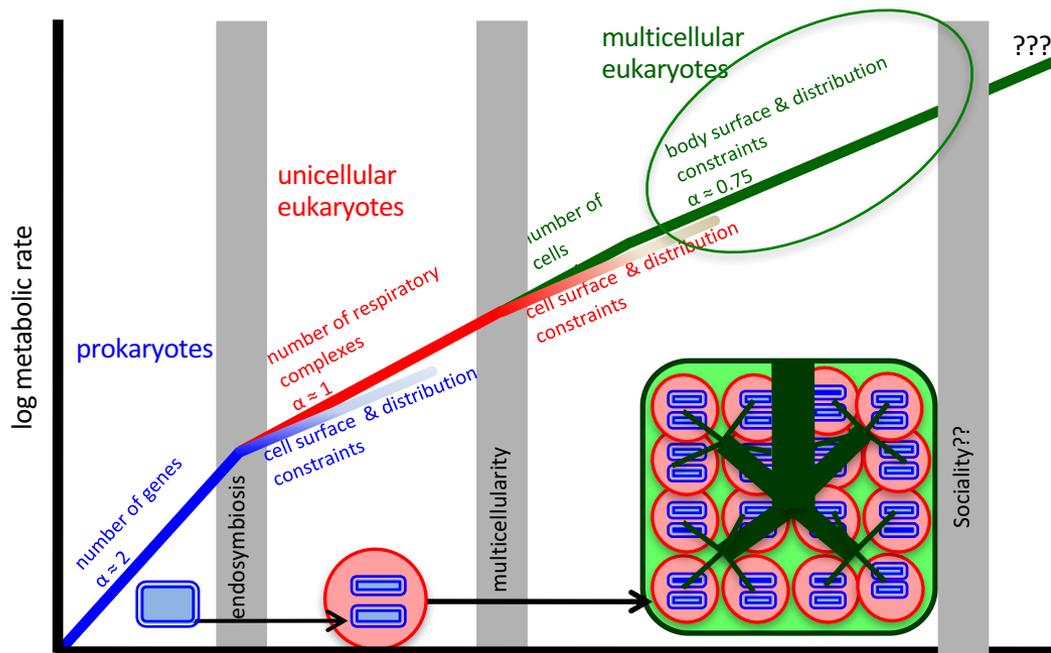
Scaling intercepts and slopes shift after evolutionary innovations



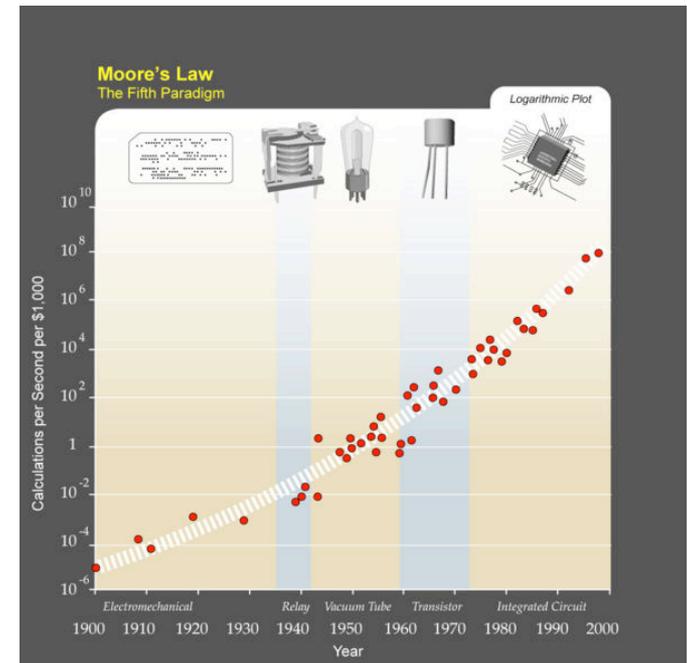
Delong et al PNAS 2010

Scaling intercepts and slopes shift after evolutionary & technological innovations

- Innovations in chip design mimic innovation in the evolution of bacteria
- Single-core chip scaling mimics unicellular protists (single cells with a nucleus)
- Multi-core chips echo the transition to multicellularity
- Computer scaling deviates from biological scaling in important ways
 - Decentralized designs dominate in the transition to sociality



Delong et al PNAS 2010



Kurzweil Singularity

Take home messages

- Approximate $\frac{3}{4}$ power scaling is ubiquitous in biology
- Scaling patterns when animals aggregate in social groups are similar to scaling patterns when cells aggregate in bodies (with notable exceptions)
- Analogous designs in microprocessor interconnect and cardiovascular network
 - Fractal branching to terminal service volumes
- Differences between scaling of energy & information
 - Information can be copied; energy can not; reduces energy dissipated to transmit a bit
 - Decentralization through communication locality (Rent's rule)
- $\frac{3}{4}$ scaling (with curvature) arises from
 - Minimizing energy dissipation and delivery time
 - matching supply and demand in the last mile for a variety of network topologies
- Network scaling framework predicts properties of engineered systems & offers a new perspective on societal infrastructure and human ecology