Logistics

- Current Reading:
  - Mitchell, *Complexity: A Guided Tour*, Chapter 1
  - Holland, *Complex Adaptive Systems*
- *"Advanced" Readings*
- Projects: Project 1 will be assigned on Monday along with your team assignments
- If you were not here last time go to office hours to fill out the intake survey and skills sheet (see Bianca)

Logistics

- Current Assignment:
  - Sign up for the mailing list.
  - Create an overleaf.com account (latex).
  - Create a github.com account (code).

- Project Format

Dynamical Systems
Lecture 2

Ver. 1.1
Change log: fixed typos on slides 34-35.

Dynamical Systems

Dynamical systems are defined by equations that describe the evolution of the system over time.

(Evolution just means change over time.)

Dynamical Systems

If we are lucky we can describe the evolution of the whole system just by knowing the current state. But we often need to know several things about the current state: i.e. its derivative.

These equations are either *differential equations* or *difference equations*.

Differential Equation

Describes the evolution of the system when that evolution depends on the rate of change of a variable. Differential equations work with *continuous* systems.

E.g models of pendulums and planetary orbits both depend on knowing the position, velocity, and acceleration.

Differential Equation

A differential equation might look like:

$y'(x, y) = f(x, y)$

The derivative of y is a function of x and y.

In general we want to find a function y such that y' is equal to f(x,y).

Differential Equation

$y'(x, y) = f(x, y)$

In general we want to find a function y

such that y' is equal to $f(x, y)$.

But there are many possible functions y

that would do the job.

To find just one we have to set an initial condition:

$y(x_0) = y_0$

Difference Equation

These equations model the evolution of a system in discrete steps. They depend only on the previous state of the system (potentially including the rate of change).

E.g. Population models over months rather than in continuous time with discrete population sizes (perhaps the Lotka-Volterra equations).

Also used to numerically solve differential equations.

Numerical Solutions: Euler's Method

- Almost all differential equations are solved numerically. In general there are no analytical solutions.
- Initial Value Problem:

$$y' = f(x, y)$$
$$y(x_0) = y_0$$

Numerical Solutions: Euler's Method

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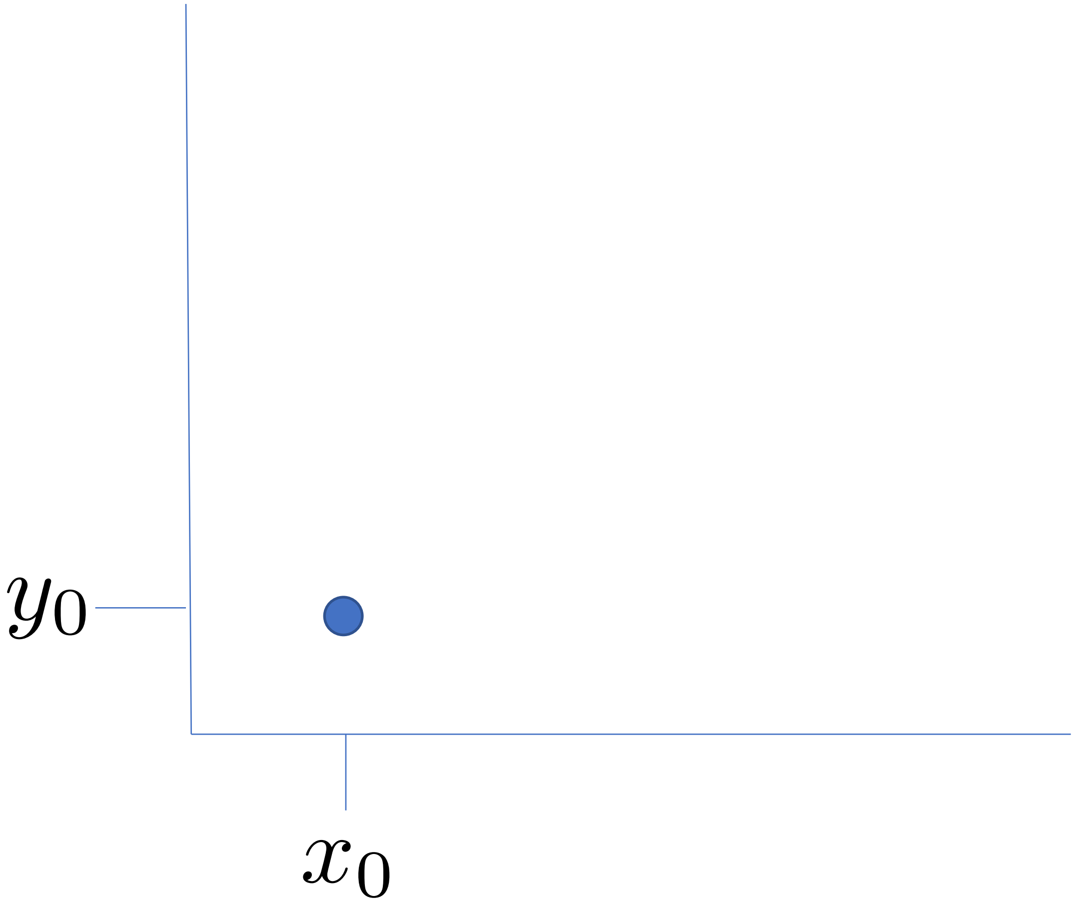
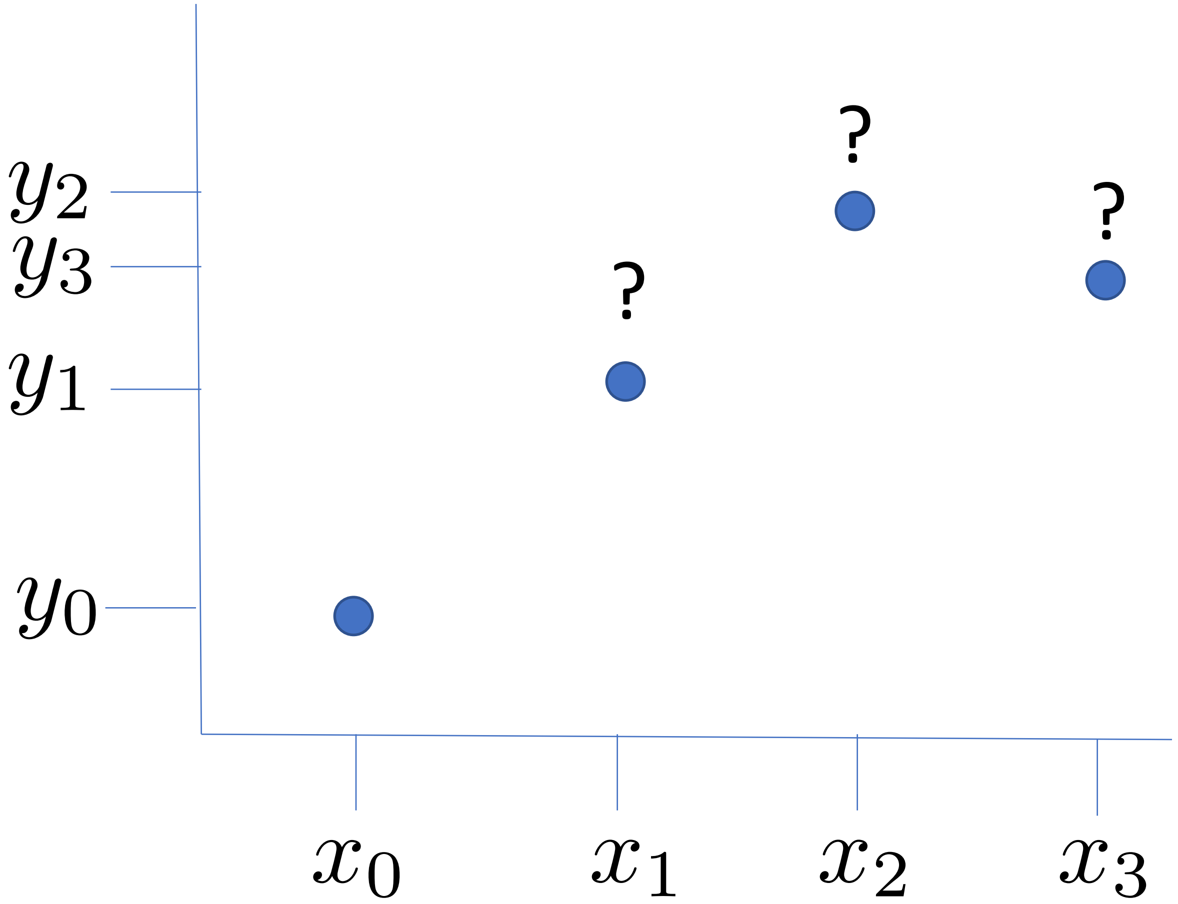


Numerical Solutions: Euler's Method

- We would like to calculate the rest of the points.
- Initial Value Problem:

$$y' = f(x, y)$$
$$y(x_0) = y_0$$

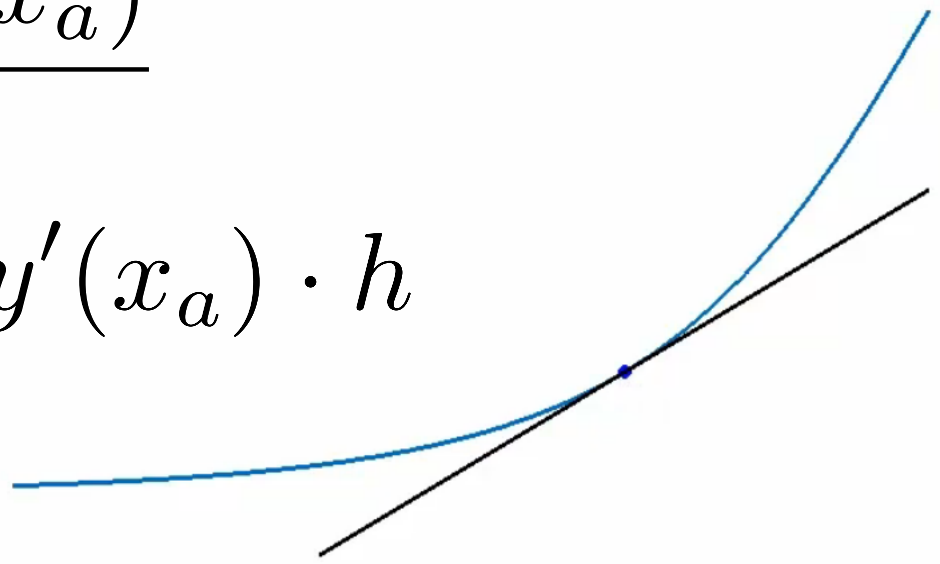


Numerical Solutions: Euler's Method

- Recall that the derivative at a point on a function is the tangent line at that point:

$$y'(x_a) \approx \frac{y(x_a + h) - y(x_a)}{h}$$

$$\therefore y(x_a + h) \approx y(x_a) + y'(x_a) \cdot h$$



Numerical Solutions: Euler’s Method

$$y'(x_a) \approx \frac{y(x_a + h) - y(x_a)}{h}$$

$$\therefore y(x_a + h) \approx y(x_a) + y'(x_a) \cdot h$$

Each point is separated by h. So we can rewrite this as:

$$y_n \approx y_{n-1} + y'_{n-1} \cdot h$$

... and now we have a difference equation.

Numerical Solutions: Euler's Method

- Tangent approximation: $y_n \approx f(x_{n-1}) + f'(x_{n-1}) \cdot h$
- Initial Value Problem:

$$y' = f(x, y)$$
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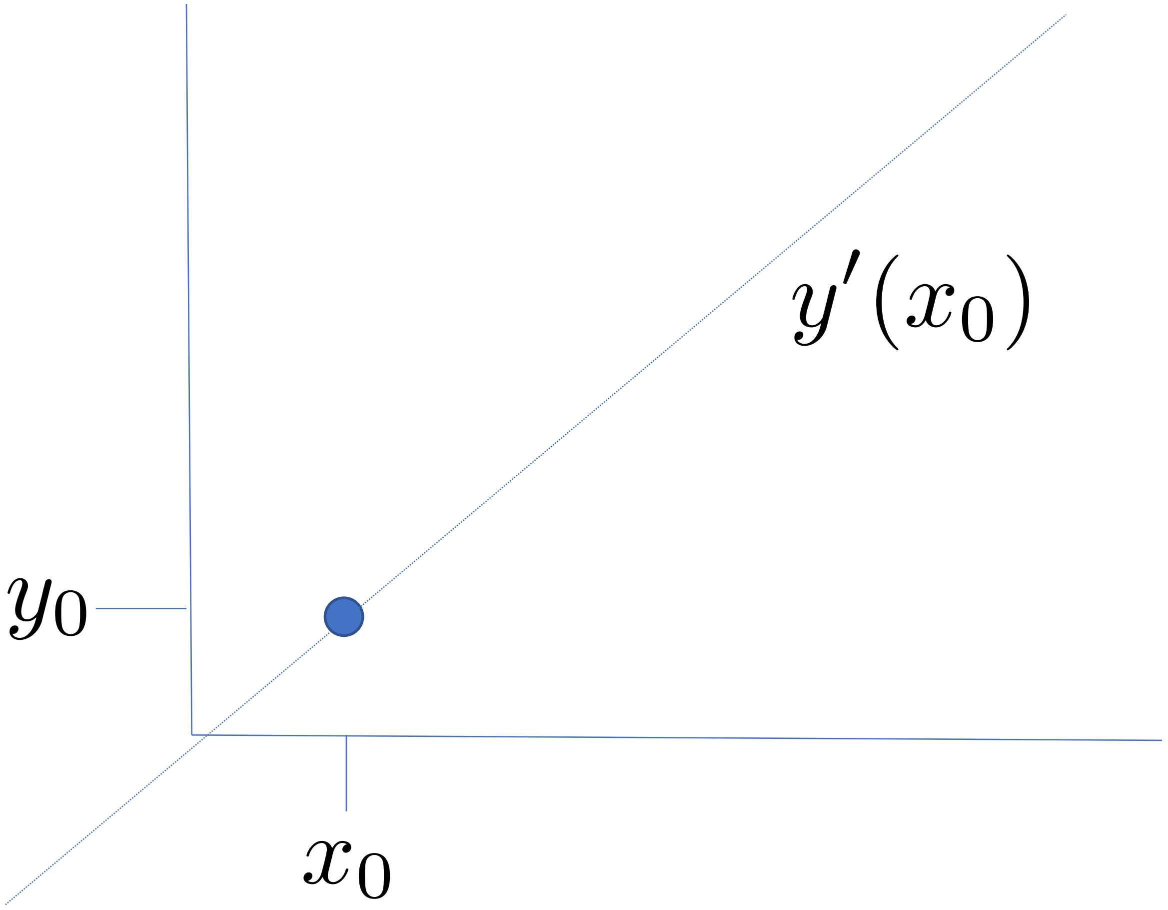

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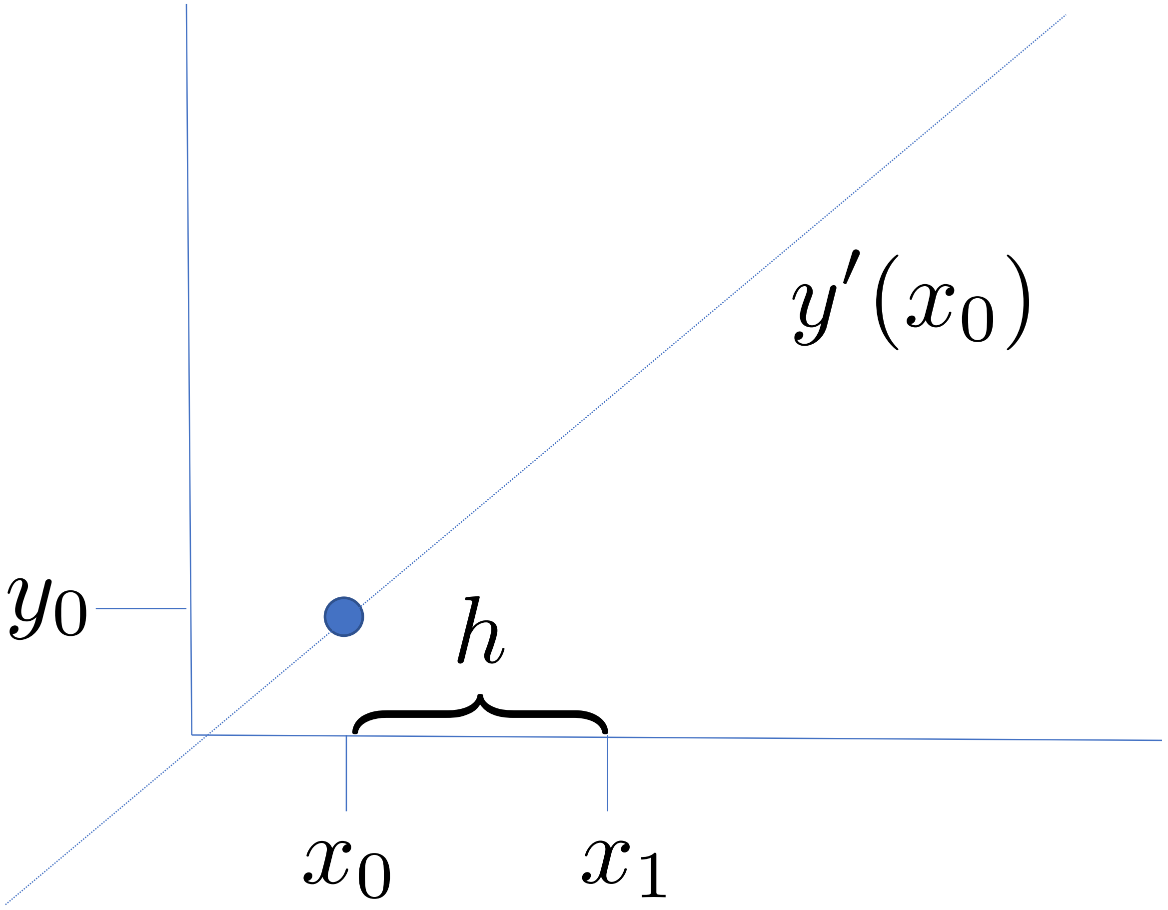

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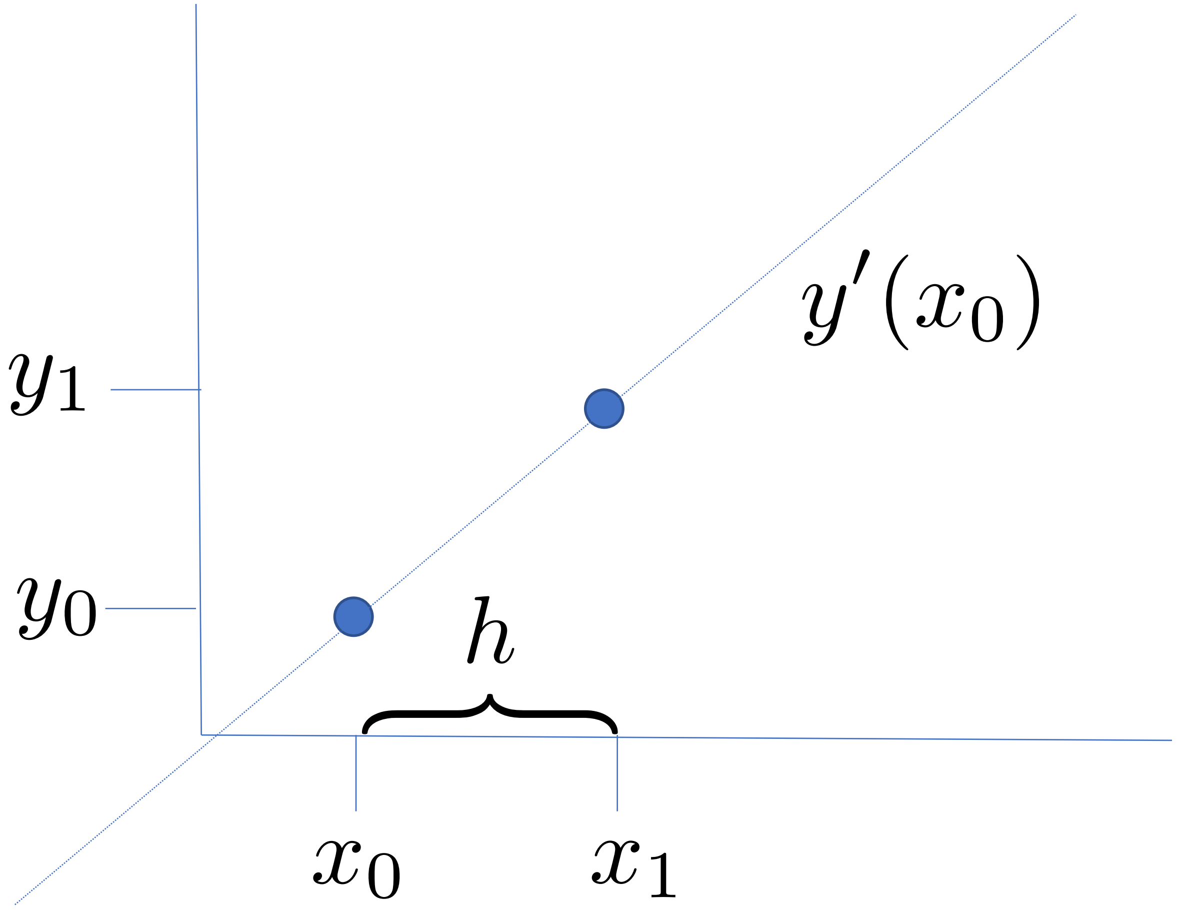

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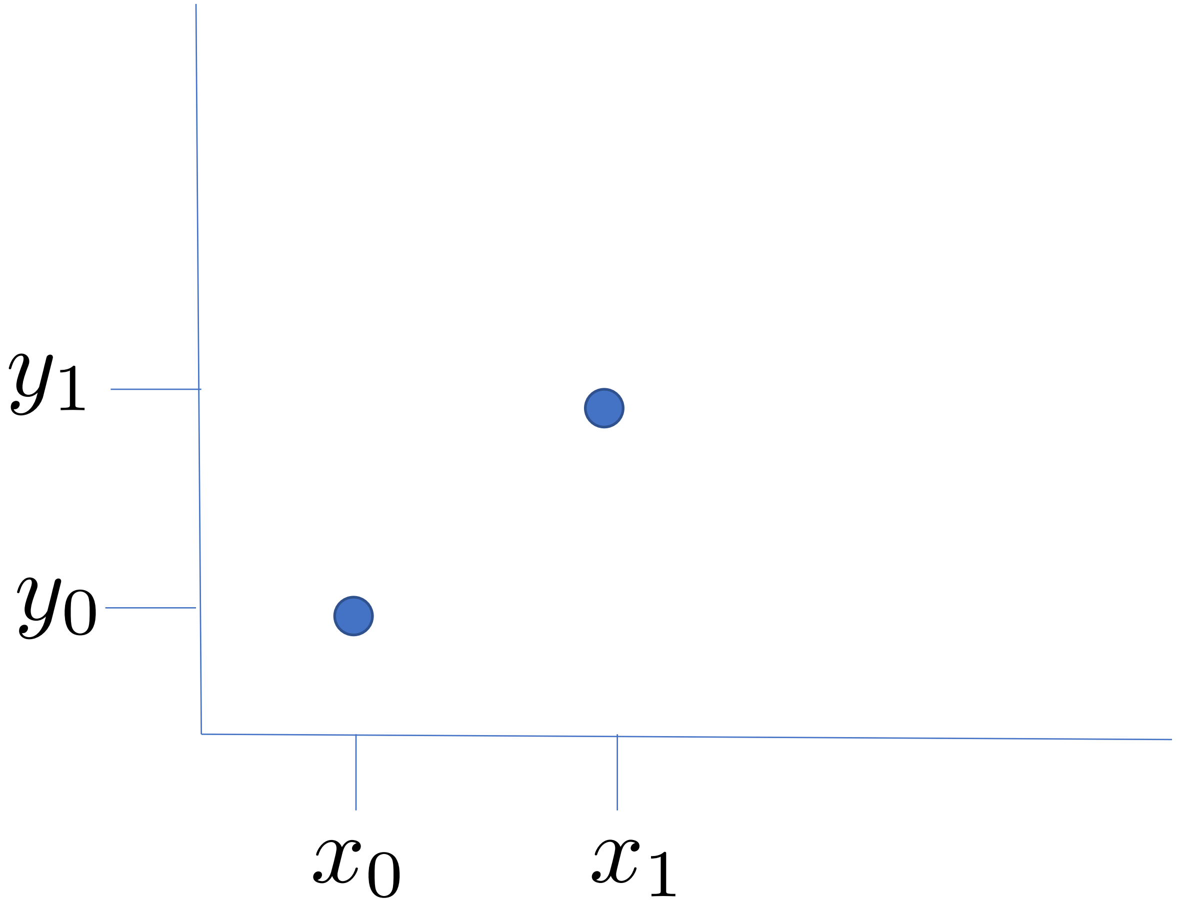

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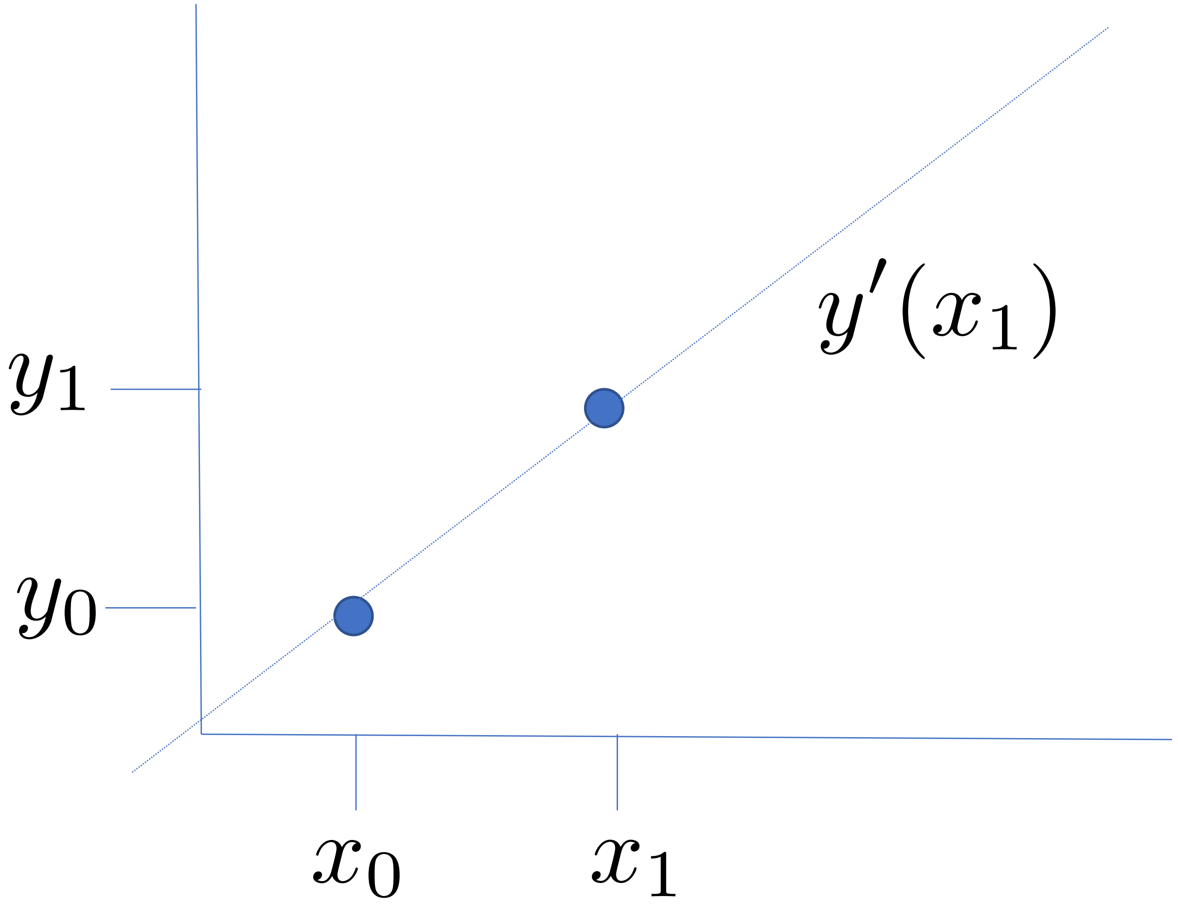

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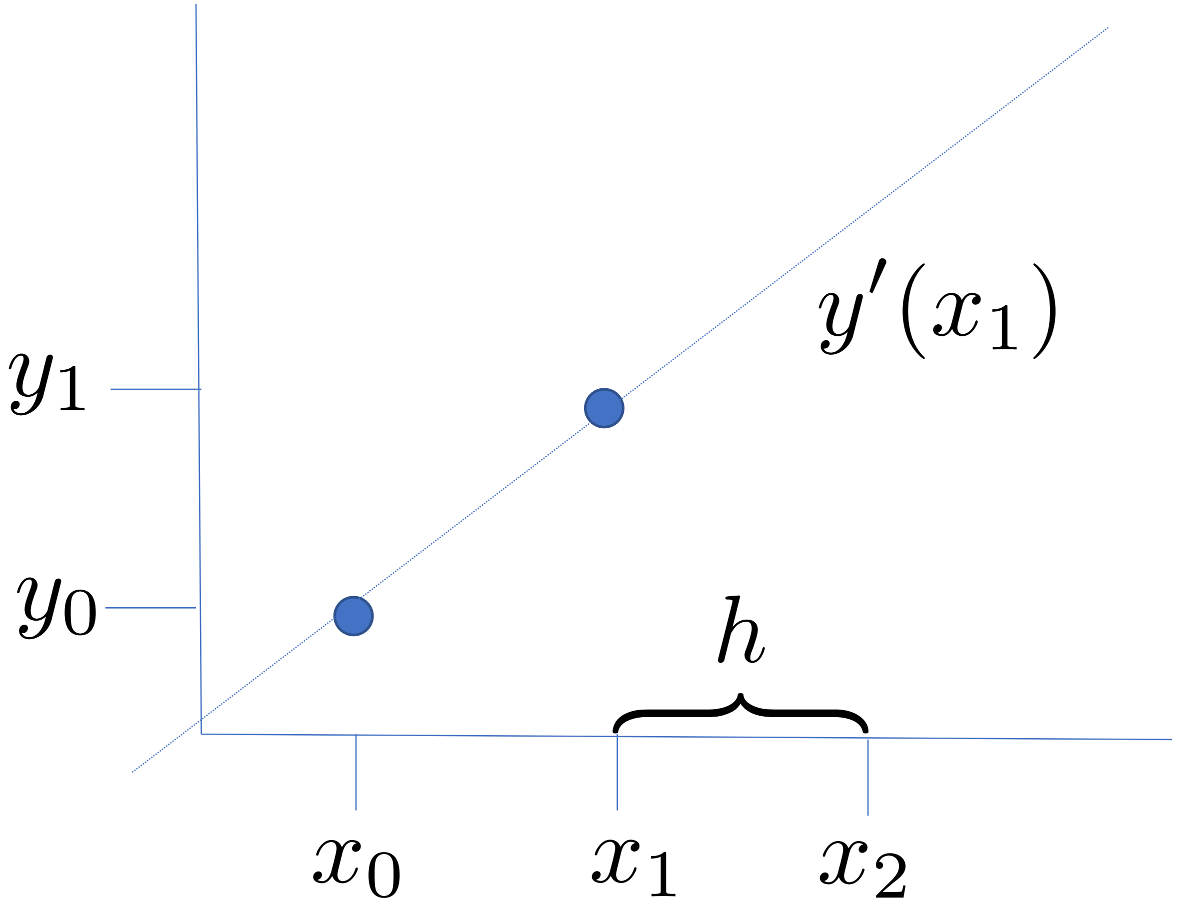

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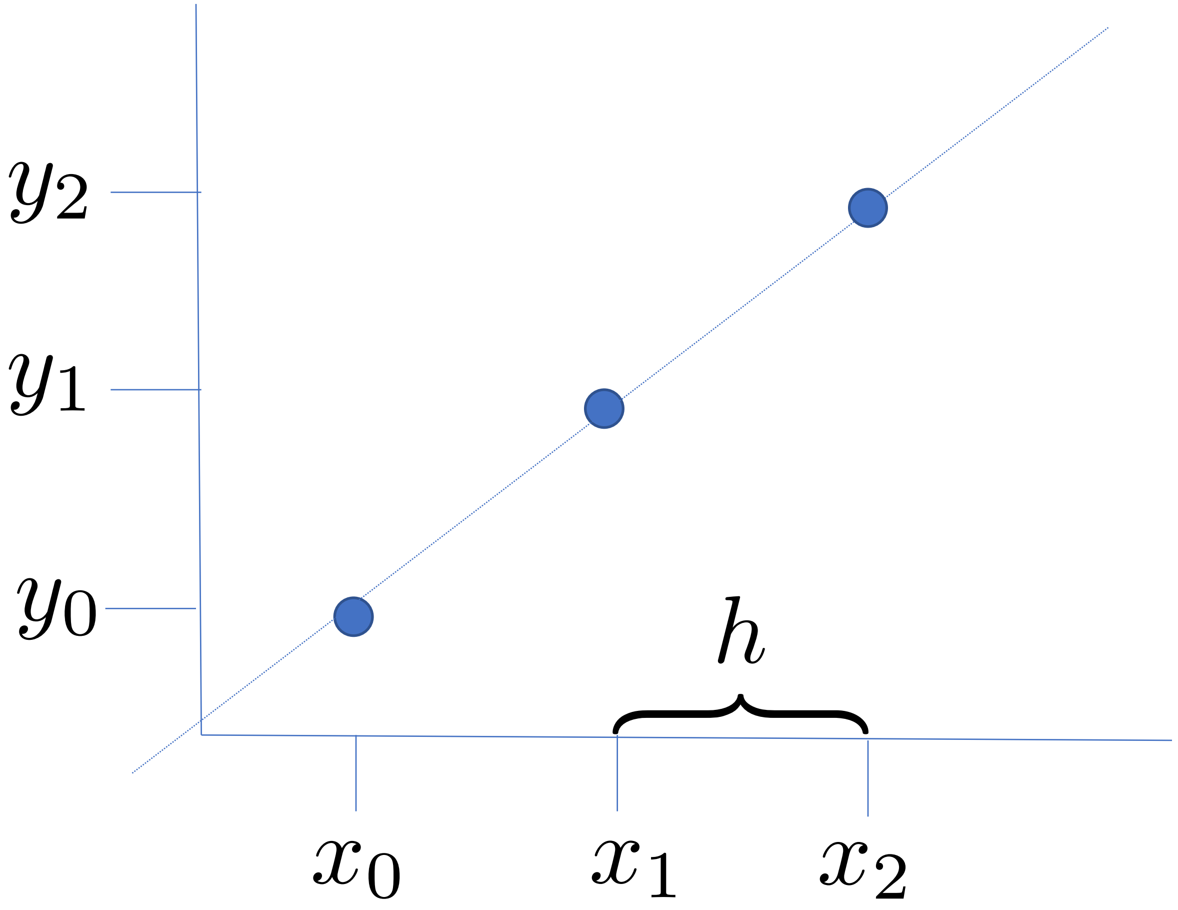

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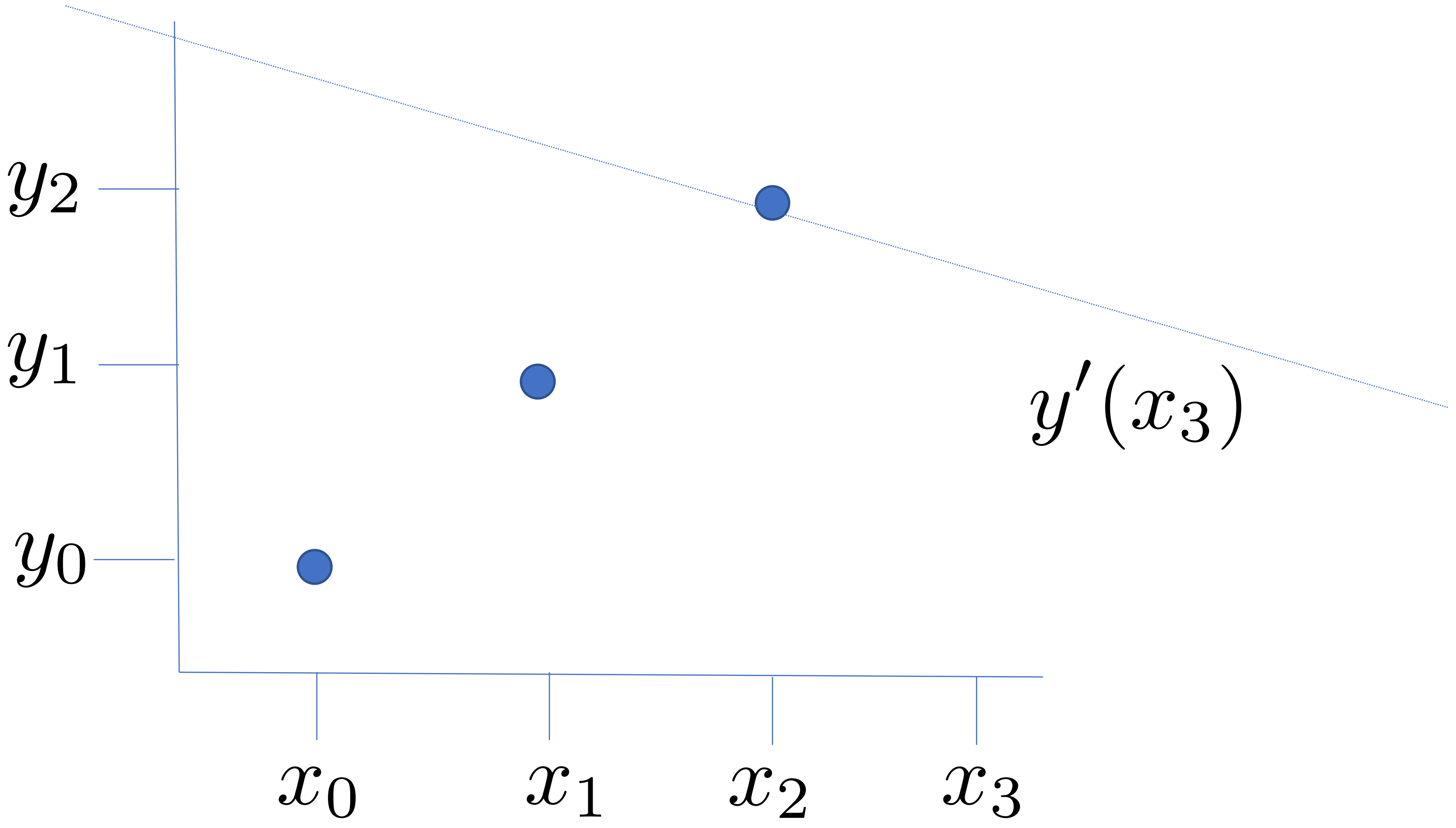

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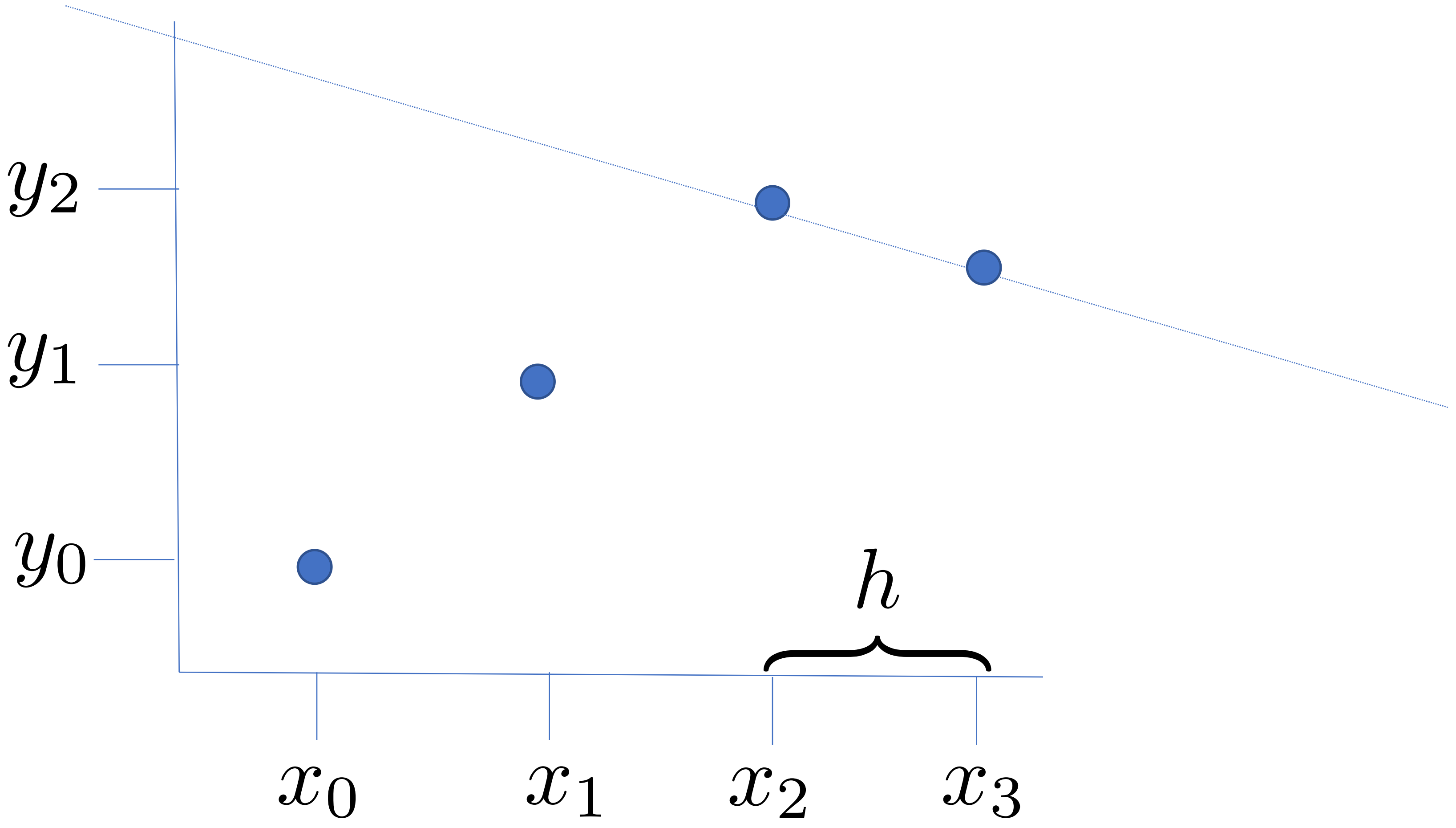

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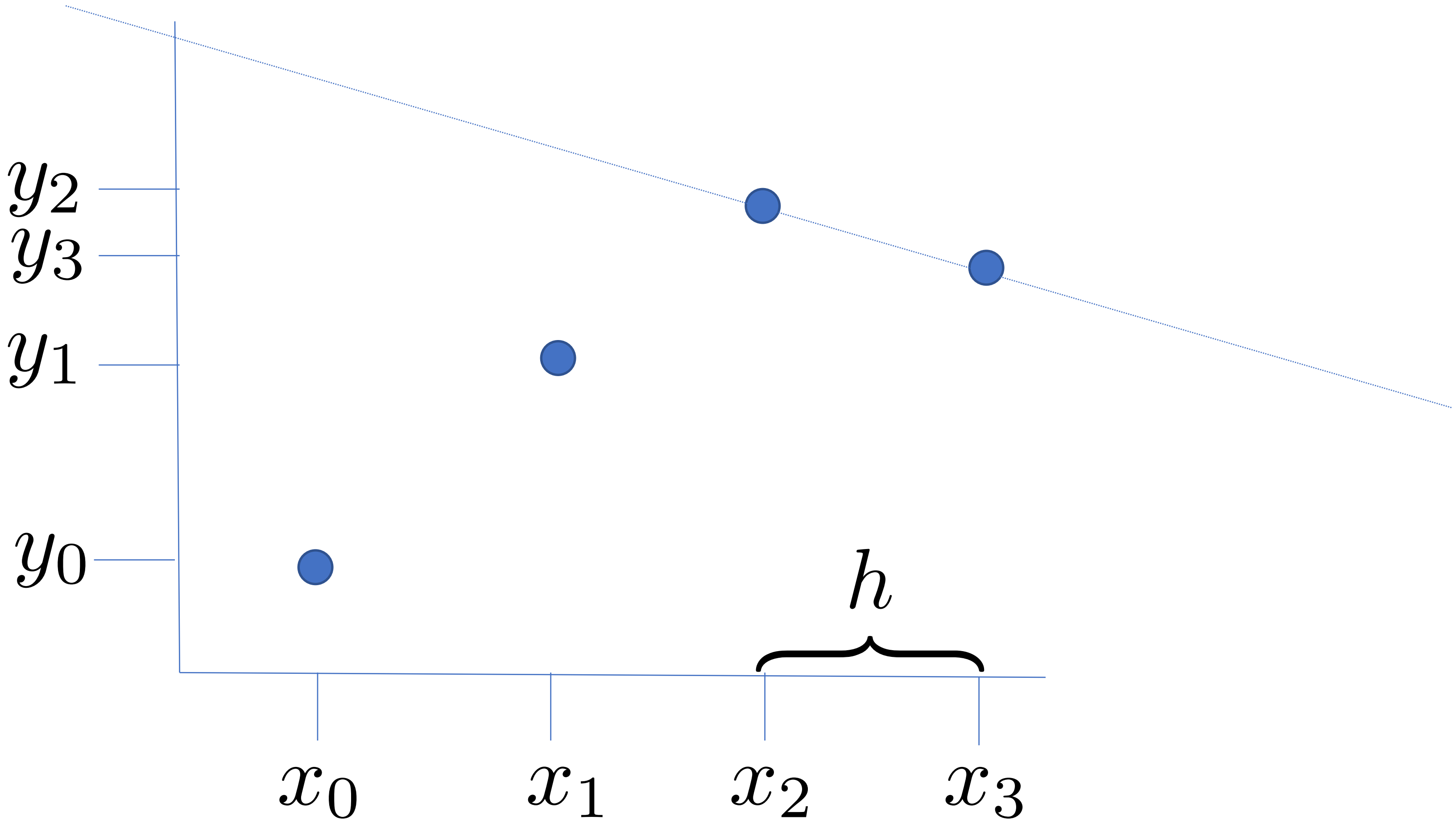

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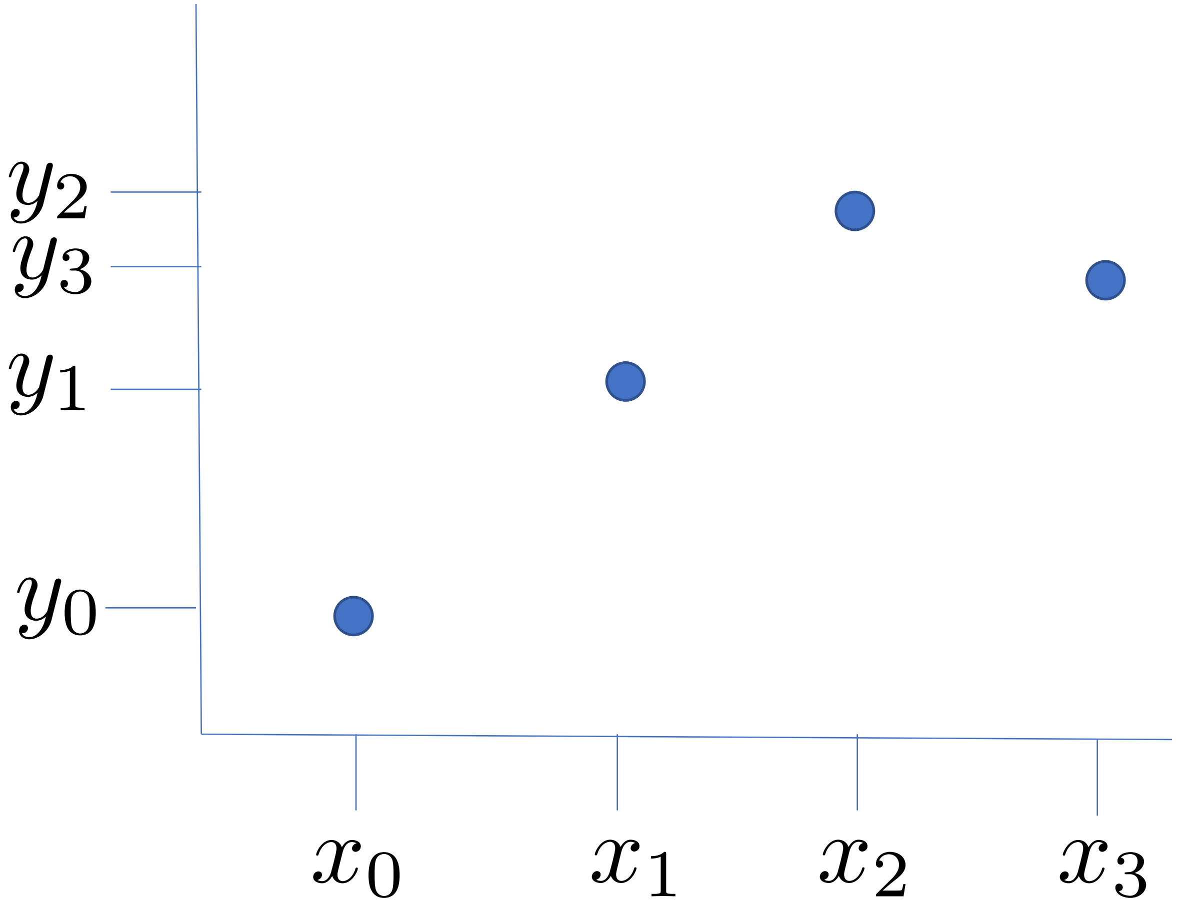

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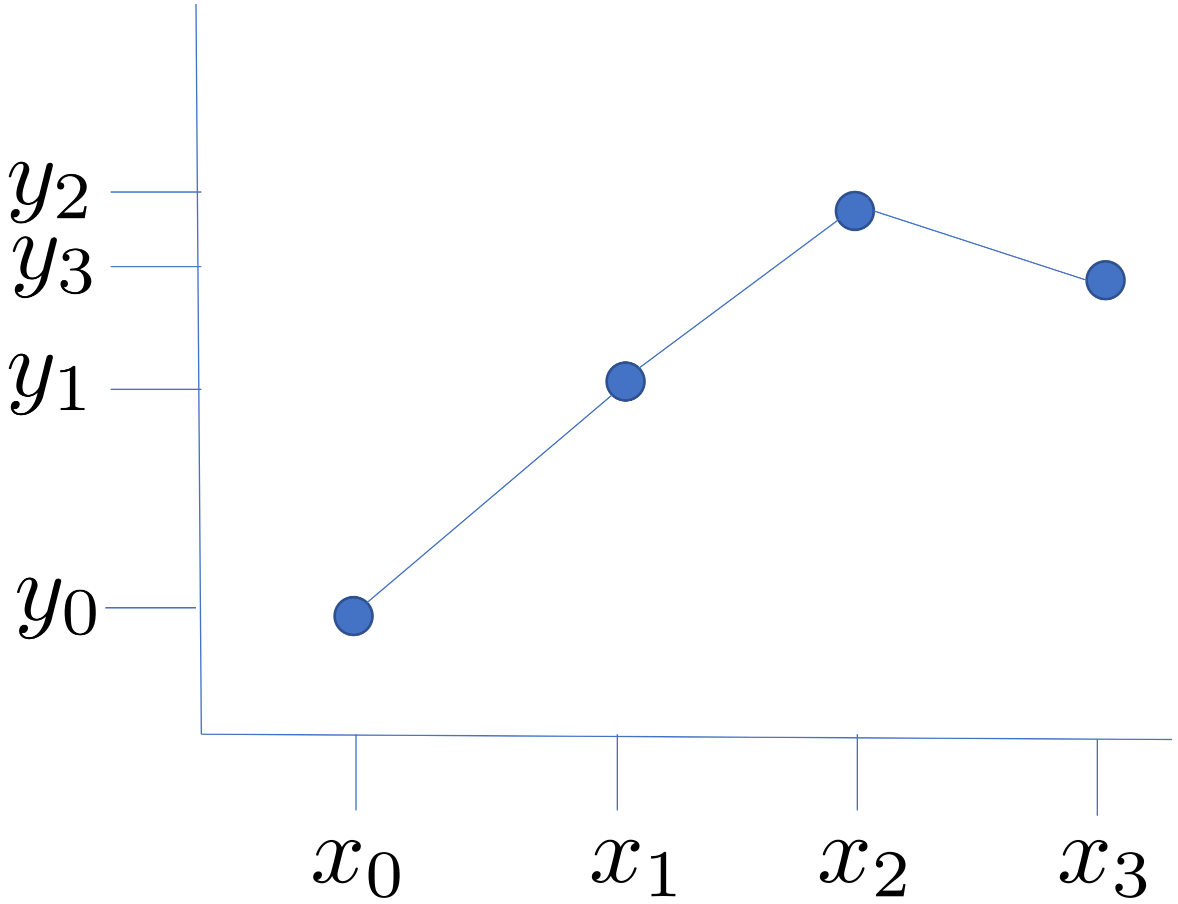

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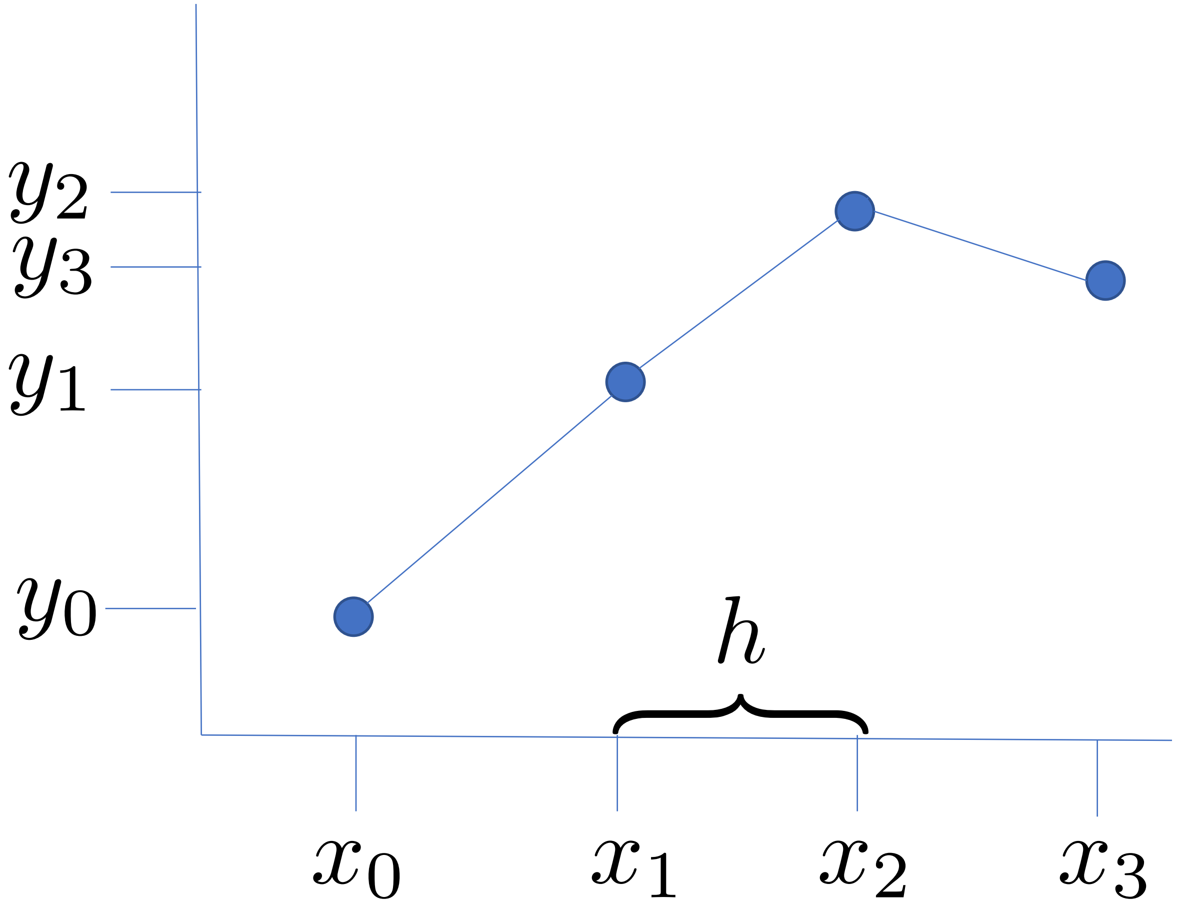


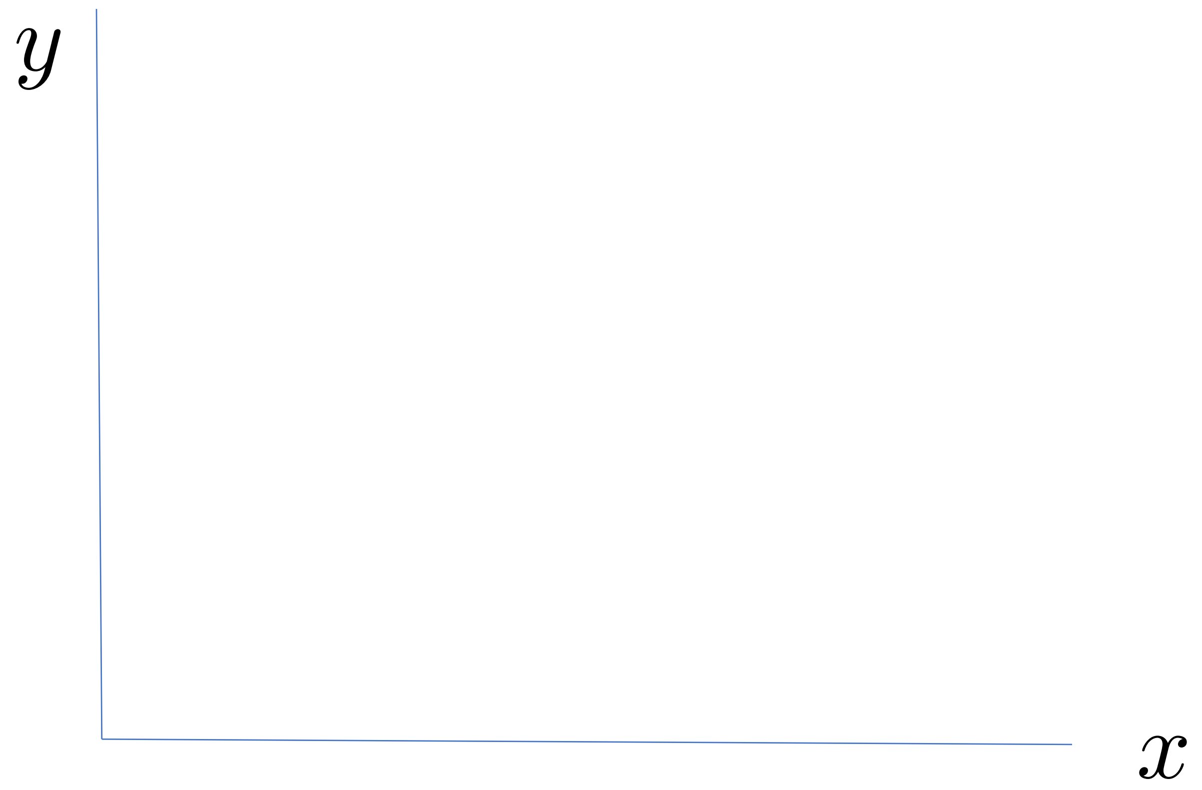
As h gets small this becomes more accurate, and we can approximate f(x).

Euler’s Method: Example

- Tangent approximation: $y_n = y_{n-1} + y'(x_{n-1}, y_{n-1}) \cdot h$
- Initial Value Problem:

$$y' = x^2 - y^2$$
$$y(1) = 0$$

Find $y(1.2)$ using $h = 0.1$.



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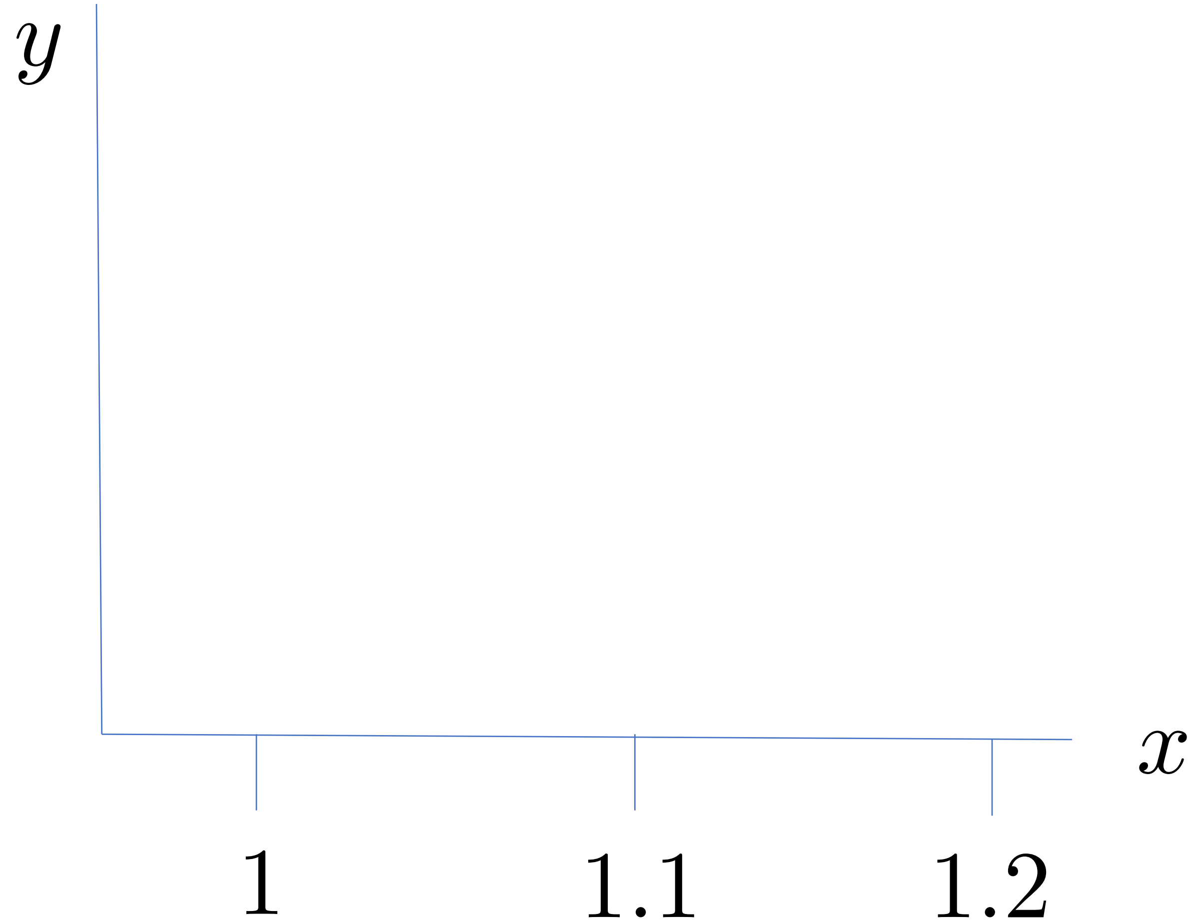

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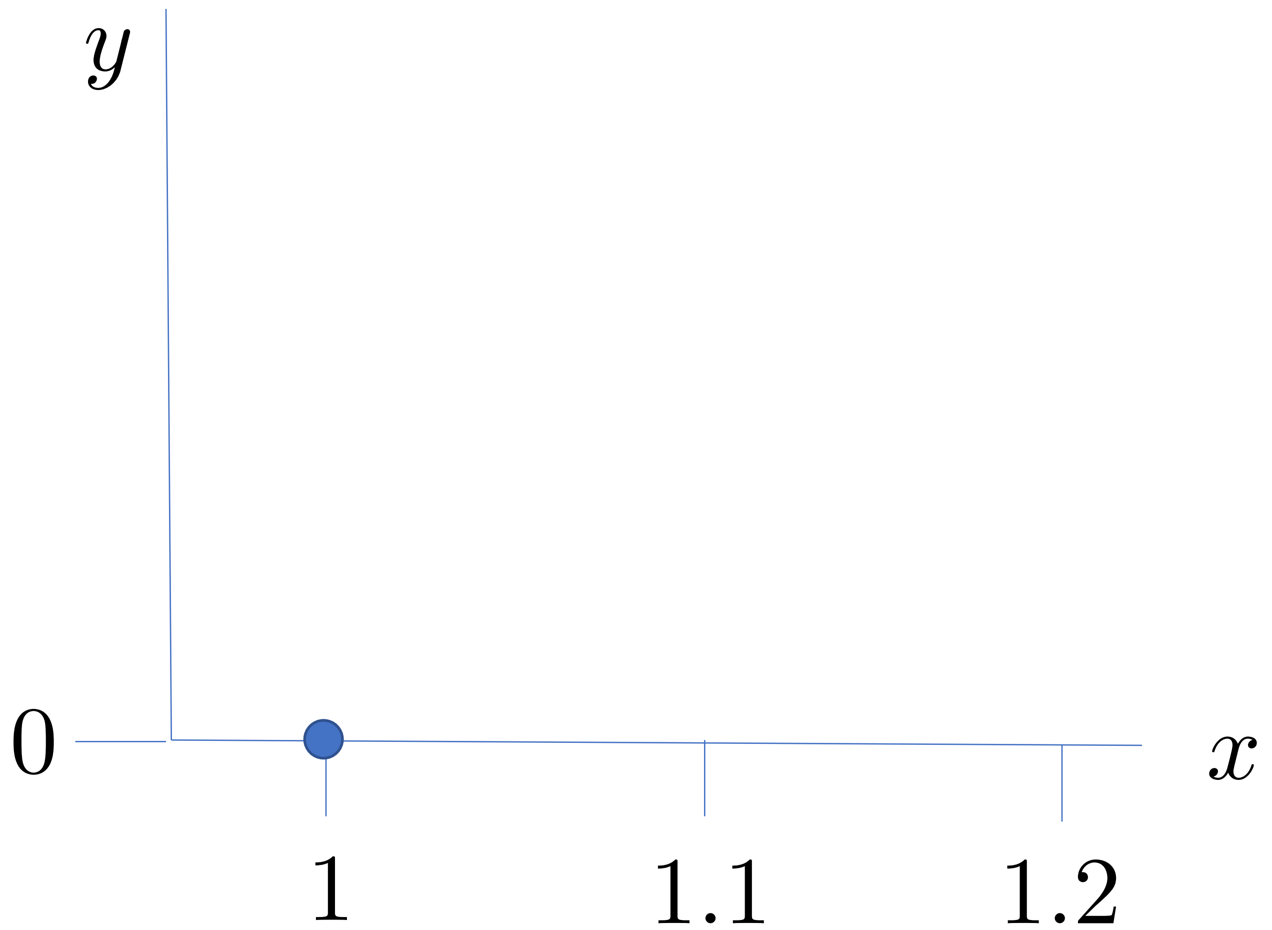

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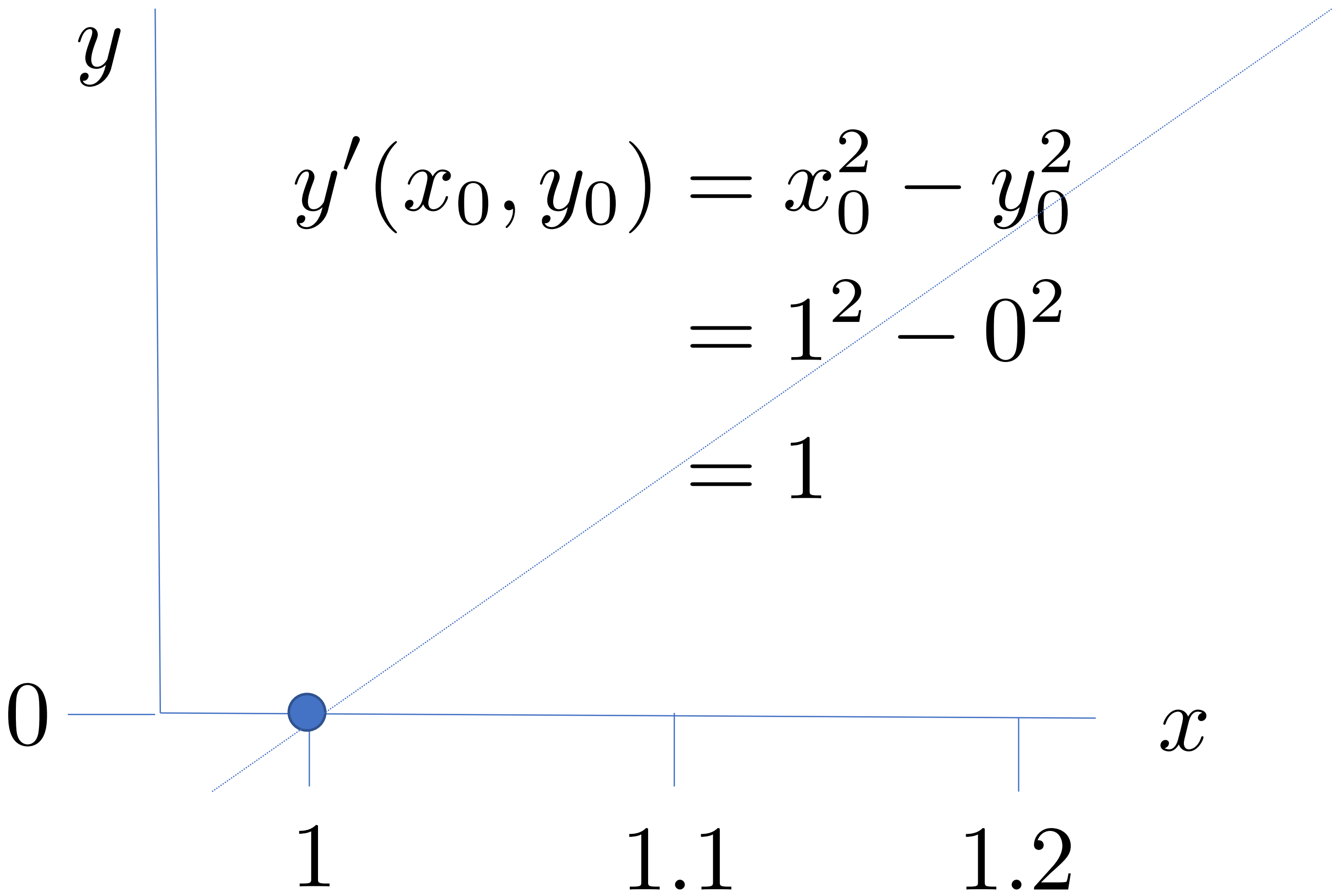

$$y'(x_0, y_0) = x_0^2 - y_0^2$$
$$= 1^2 - 0^2$$
$$= 1$$

Euler's Method: Example

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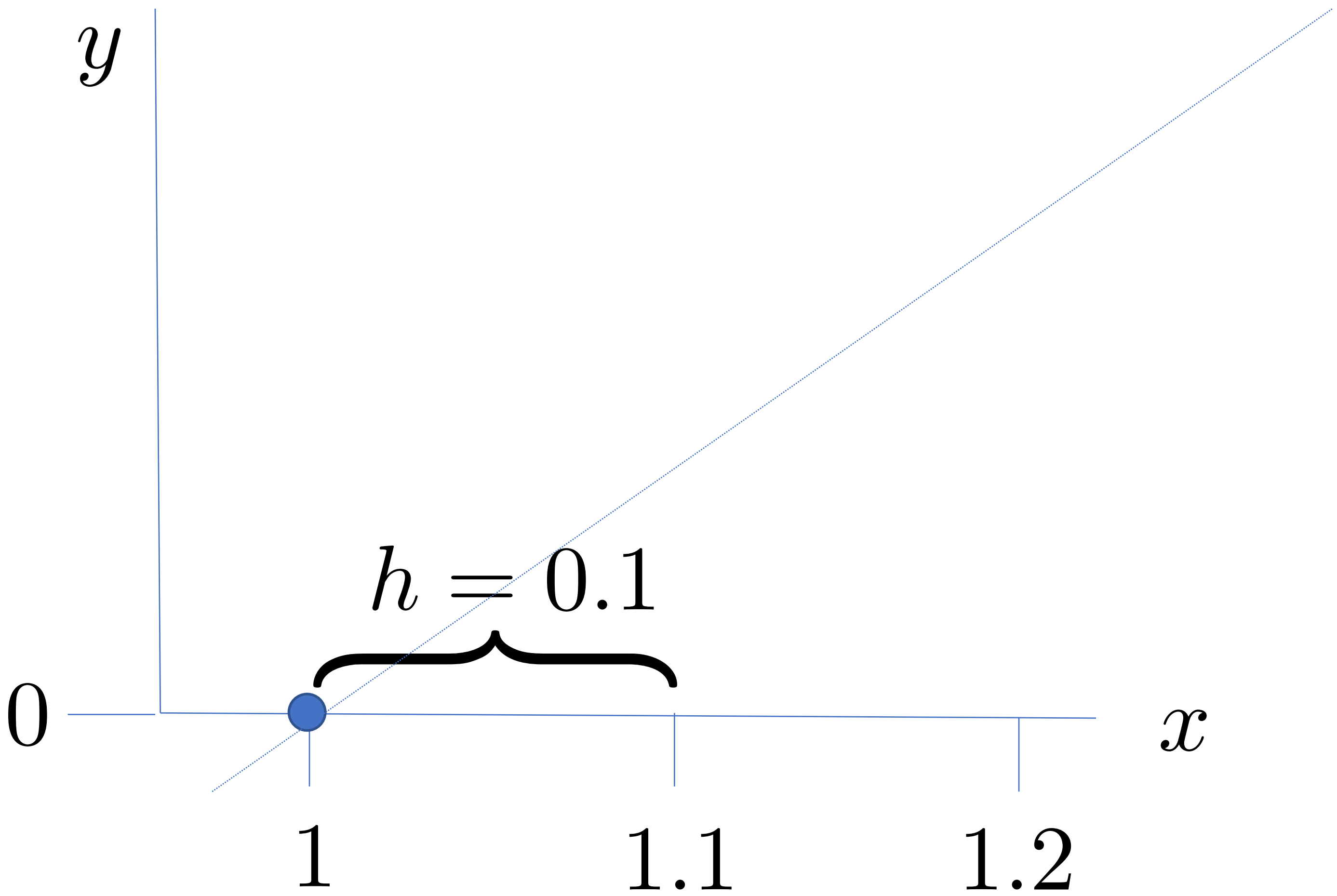

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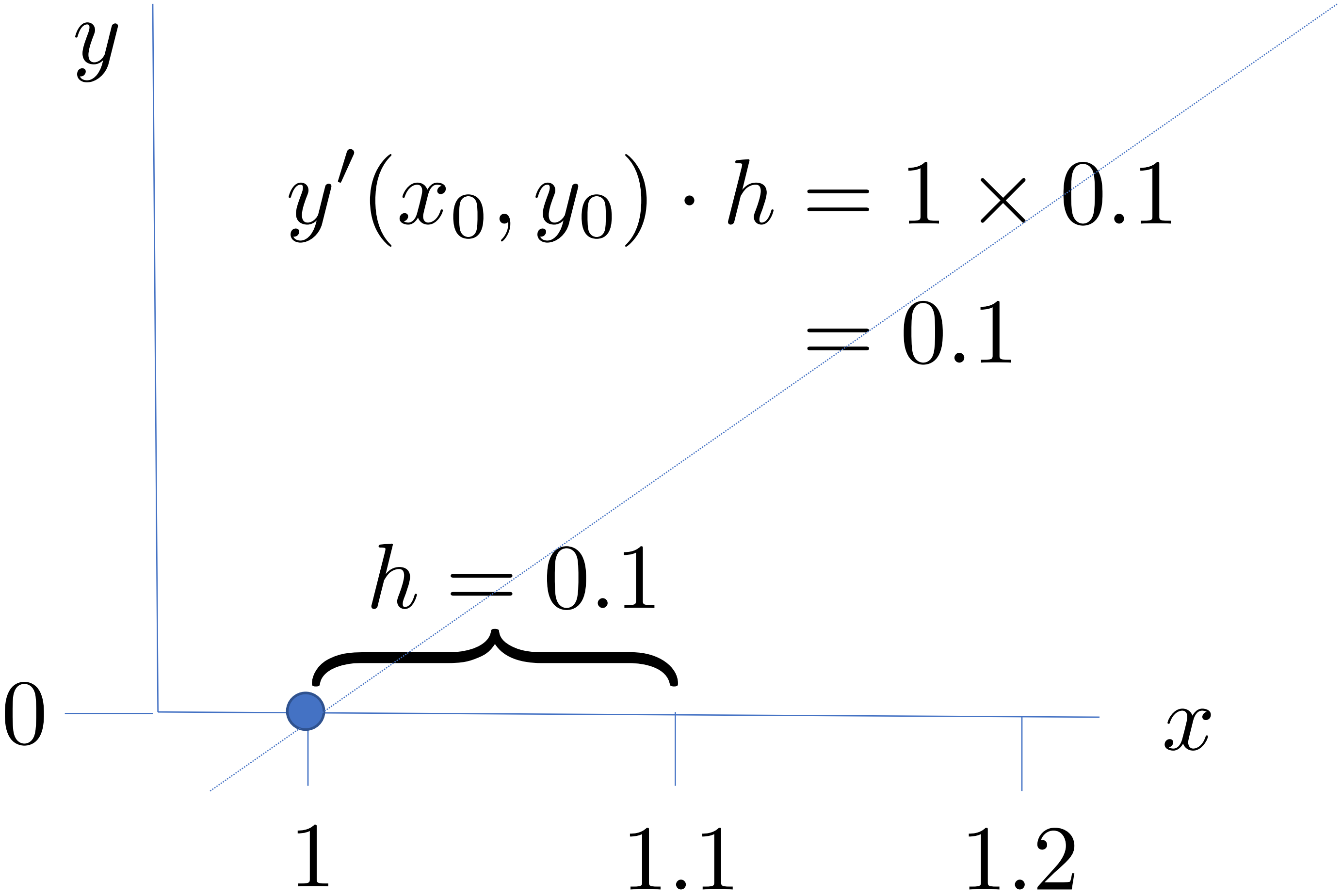

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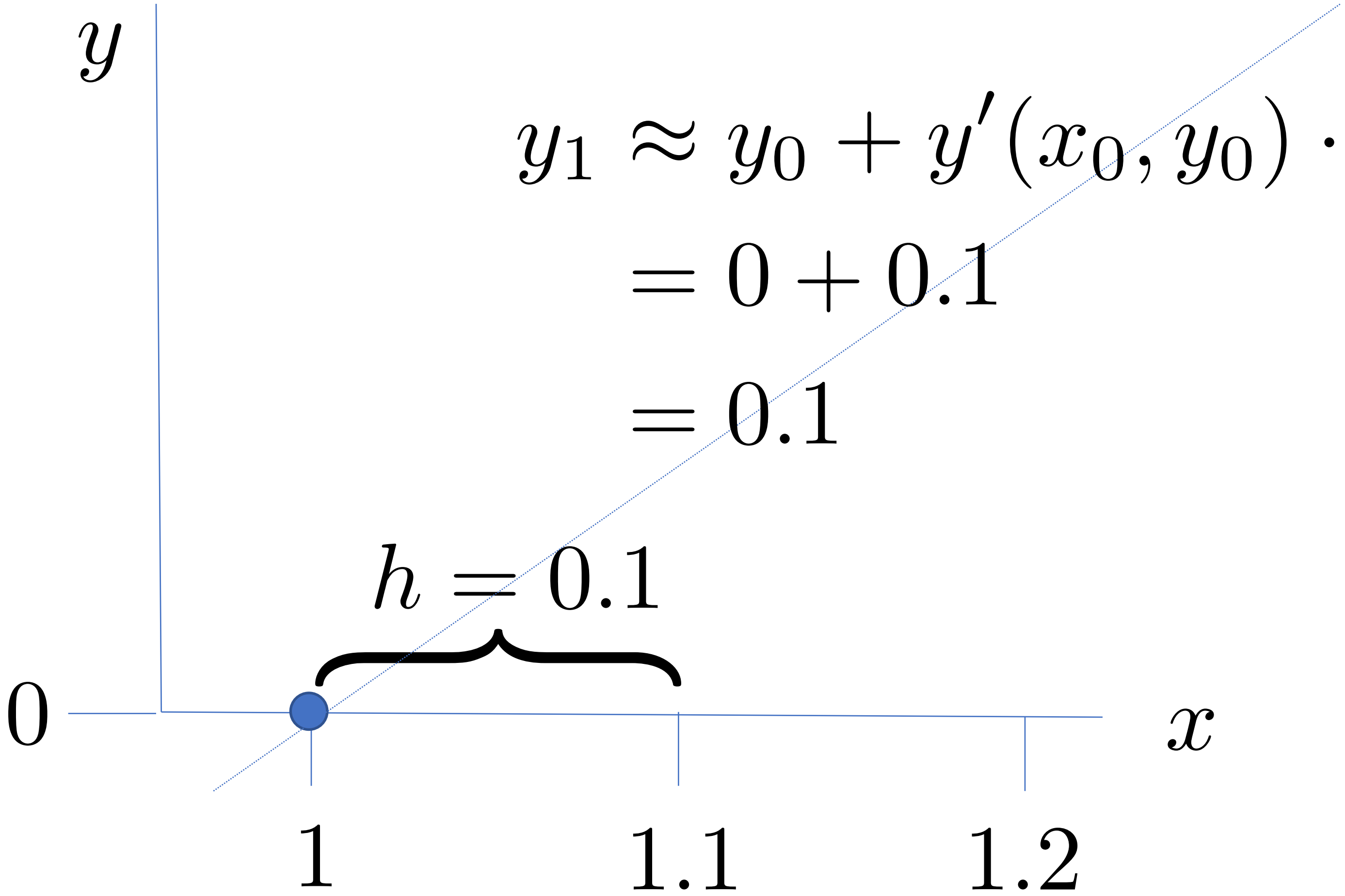
$$y(1) = 0$$

Find $y(1.2)$ using $h = 0.1$.

$$y_1 \approx y_0 + y'(x_0, y_0) \cdot h$$
$$= 0 + 0.1$$
$$= 0.1$$



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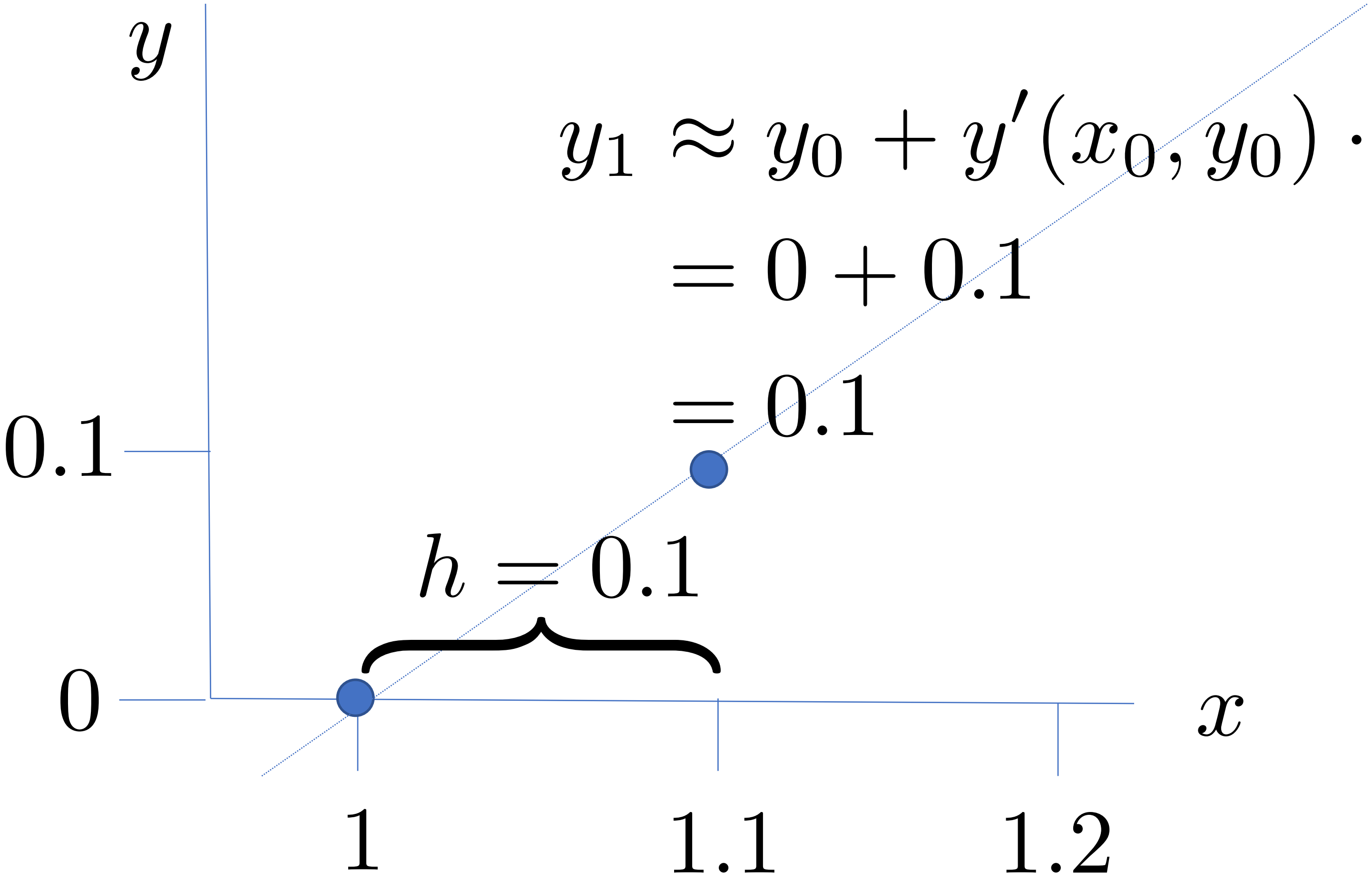

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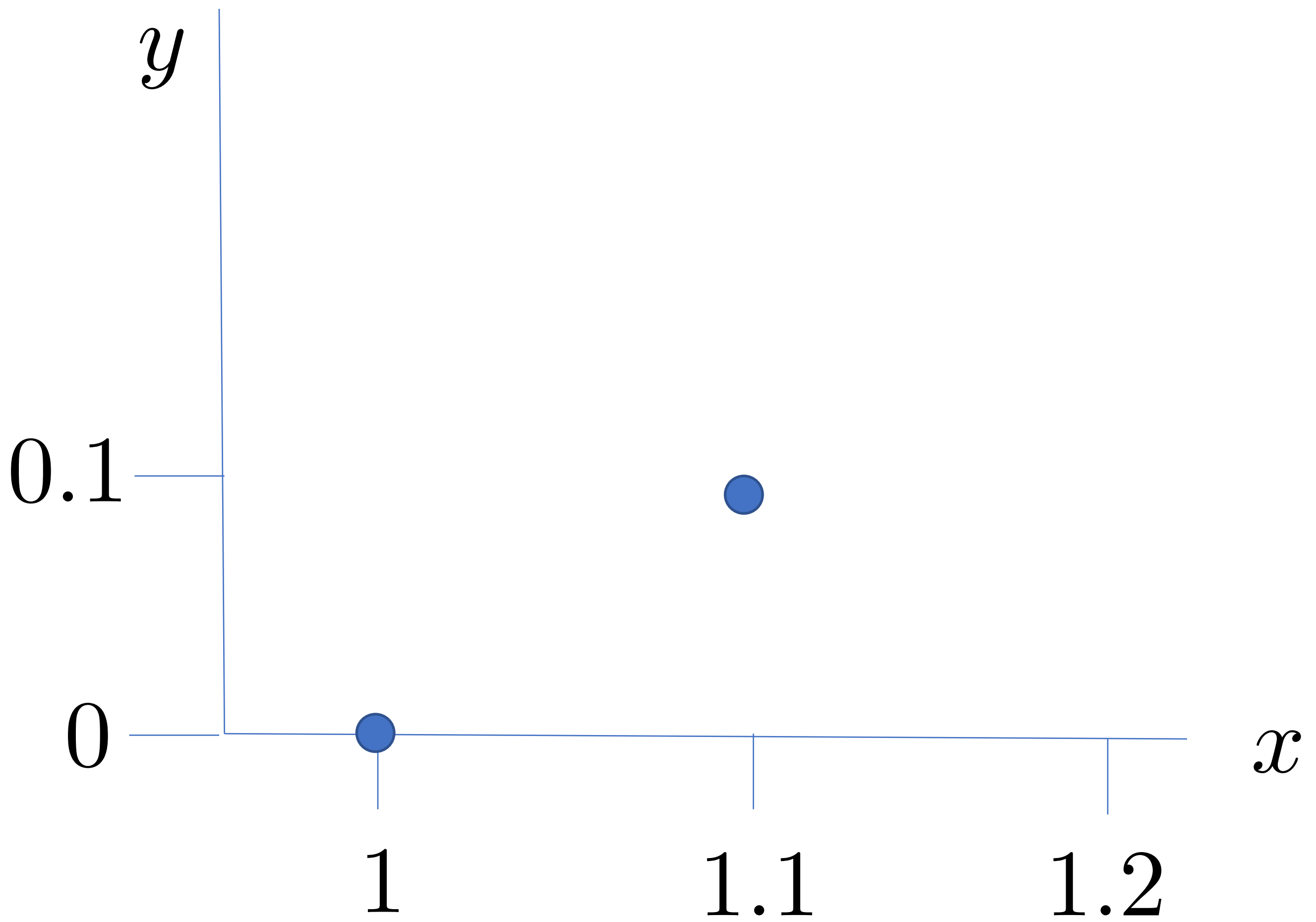

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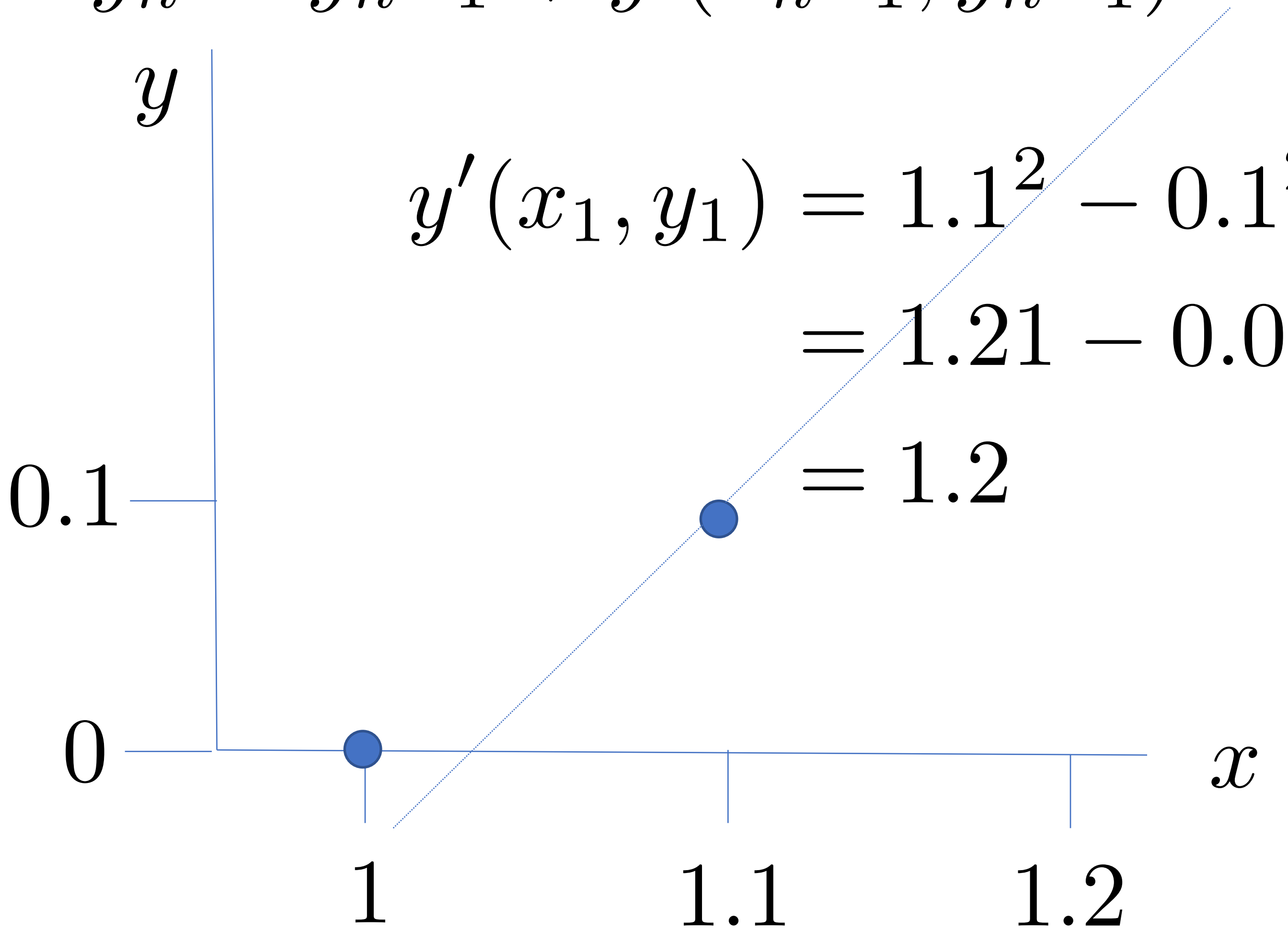

$$y'(x_1, y_1) = 1.1^2 - 0.1^2$$
$$= 1.21 - 0.01$$
$$= 1.2$$

Euler's Method: Example

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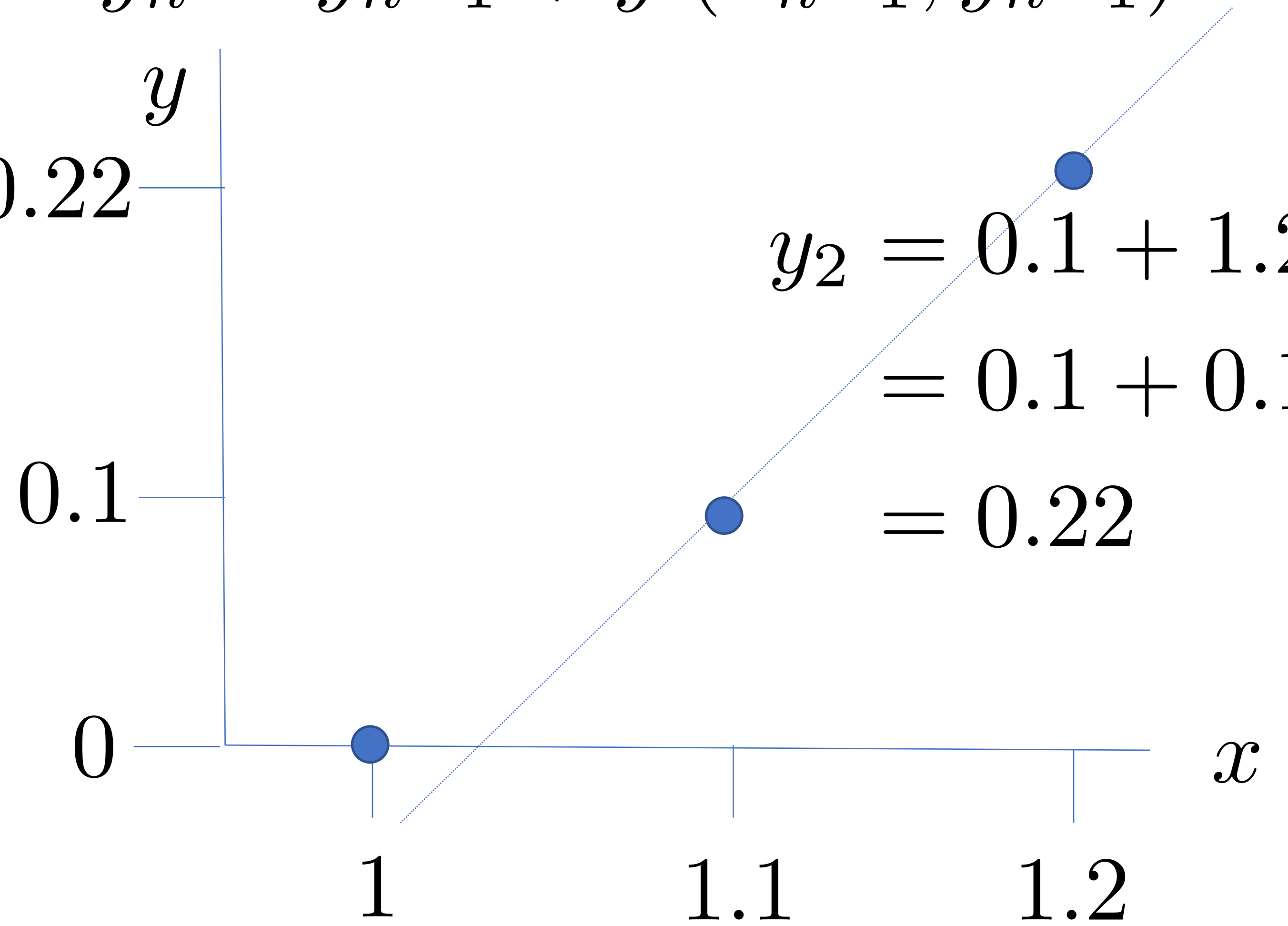
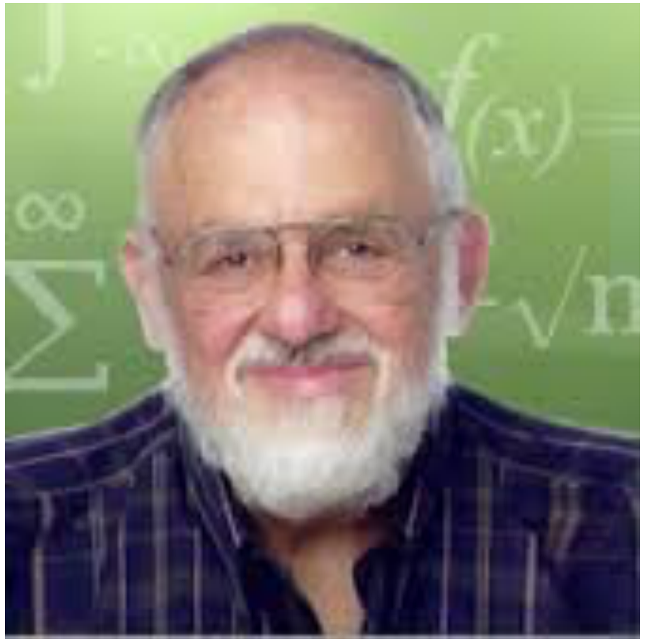
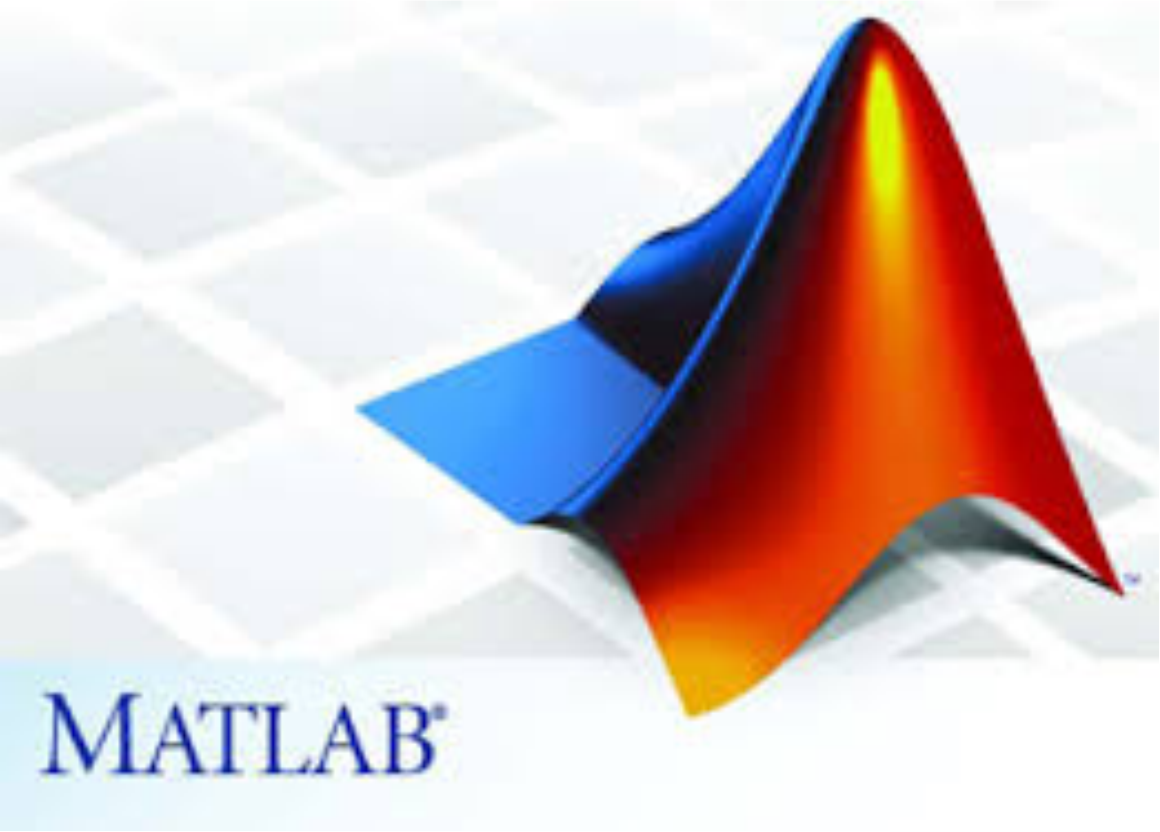


$$y_2 = 0.1 + 1.2 \times 0.1$$
$$= 0.1 + 0.12$$
$$= 0.22$$

Let's write a program to do this!





Cleve Moler
Freely available to UNM students:
http://it.unm.edu/download/

Logistic Function

The logistic map is a simple population model.

For example:

$$f(x, y) = 3y - 2y^2$$

$$y_0 = 1$$

The first term is the birth rate the second is the death rate.

Next Time

Going Deep with the Logistic Map

Nonlinear Systems of Equations:

Lotka-Voltera