Problem 1. Functions and Pigeonhole Principle

(a) Determine whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^{\frac{1}{3}}, \forall x \in \mathbb{R}$ is bijective.

(b) How many people need to be at a party to ensure that at least 3 of them have birthdays in June, 3 in July, 3 in August, or 4 in some other month.

Problem 2. Discrete Probability

Consider the probability table:

	red	green
ripe	0.36	0.06
unripe	0.04	0.54

$X = \{\text{ripe}, \text{unripe}\}$
$Y = \{\text{red}, \text{green}\}$

(a) Calculate $P(\mathrm{X} = \text{ripe} \cap \mathrm{Y} = \text{green})$

(b) Calculate $P(\mathrm{X} = \text{ripe} \mid \mathrm{Y} = \text{green})$

(c) What is the probability of getting at least 4 heads out of 10 tosses of a coin.

Problem 3. Discrete Probability: Bayes' Theorem and the Binomial Distribution

Consider two boxes each containing red and green marbles. One box contains three times as many red marbles as green. The other box contains three times as many green marbles as red. Suppose we choose one of these boxes at random. From this box we select five marbles at random, replacing each marble after it has been selected. The result is that we find 4 green marbles and one red. What is the probability that we were using the box with mainly green marbles?

Problem 4. Recurrence Relations

(a) Find the closed form for this relation and prove it is true with induction.
$a_n = a_{n-1} + 3$
$a_1 = 2$

(b) Give the solution in asymptotic notation. That is give g(n) such that $a_n \in O(g(n))$.

Problem 5. Recurrence Relations

(a) Find the closed form for this relation:
$T(n) - 4T(n-1) + 3T(n-2) = 0$
$T(0) = 0$
$T(1) = 2$

(b) Give the solution in asymptotic notation. That is give $g(n)$ such that $T(n) \in O(g(n))$.

Problem 6. Relations

Consider the relation R which is defined on $\mathbb{R}$ by a R b if $ab \leq 0$.

(a) Give an example of two real numbers related by R.

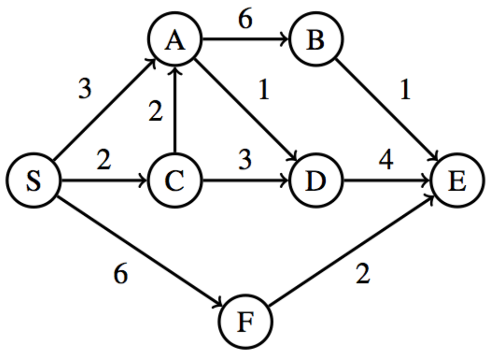
(b) Give an example of two real numbers that are not related by R.

(c) Which of the properties *reflexive*, *symmetric*, and *transitive* does R possess?

R is defined to be a relation between a and b if $a - b$ is an integer.

(d) Show the relation R is an equivalence relation.

Problem 7. Graphs and Trees



(a) Find the shortest path from S to E.

(b) Find the longest path from S to E.

(c) Treat the graph as an activity graph and find the float time for each activity.

(d) The speed of an electronic circuit is limited by its critical path. If this graph were an electronic circuit with weights being speeds what would be the limiting speed?

Problem 8. Graphs and Trees

Given a connected graph with n vertices and no cycles prove that it must have n-1 edges using structural induction.

Problem 9. Models of Computation

Draw the multigraph for a deterministic finite-state automaton that recognizes strings that start with a 1 and end with a 1.

Problem 10. Models of Computation

Write a Turing machine that given a binary number b calculates b+1.