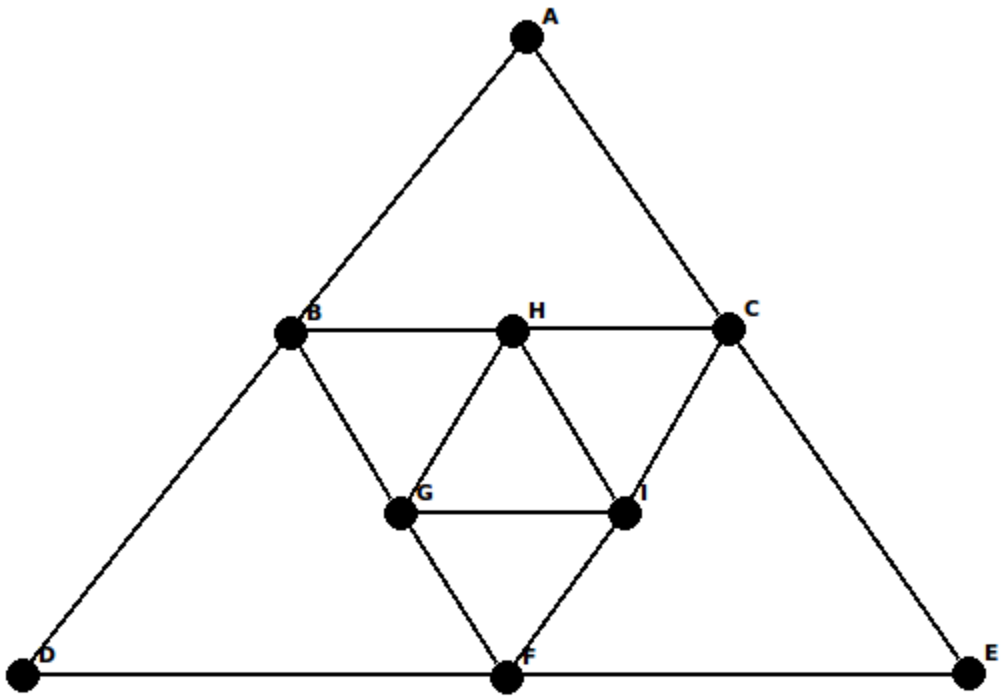
Problem 1. Connectedness and Equivalence Relations

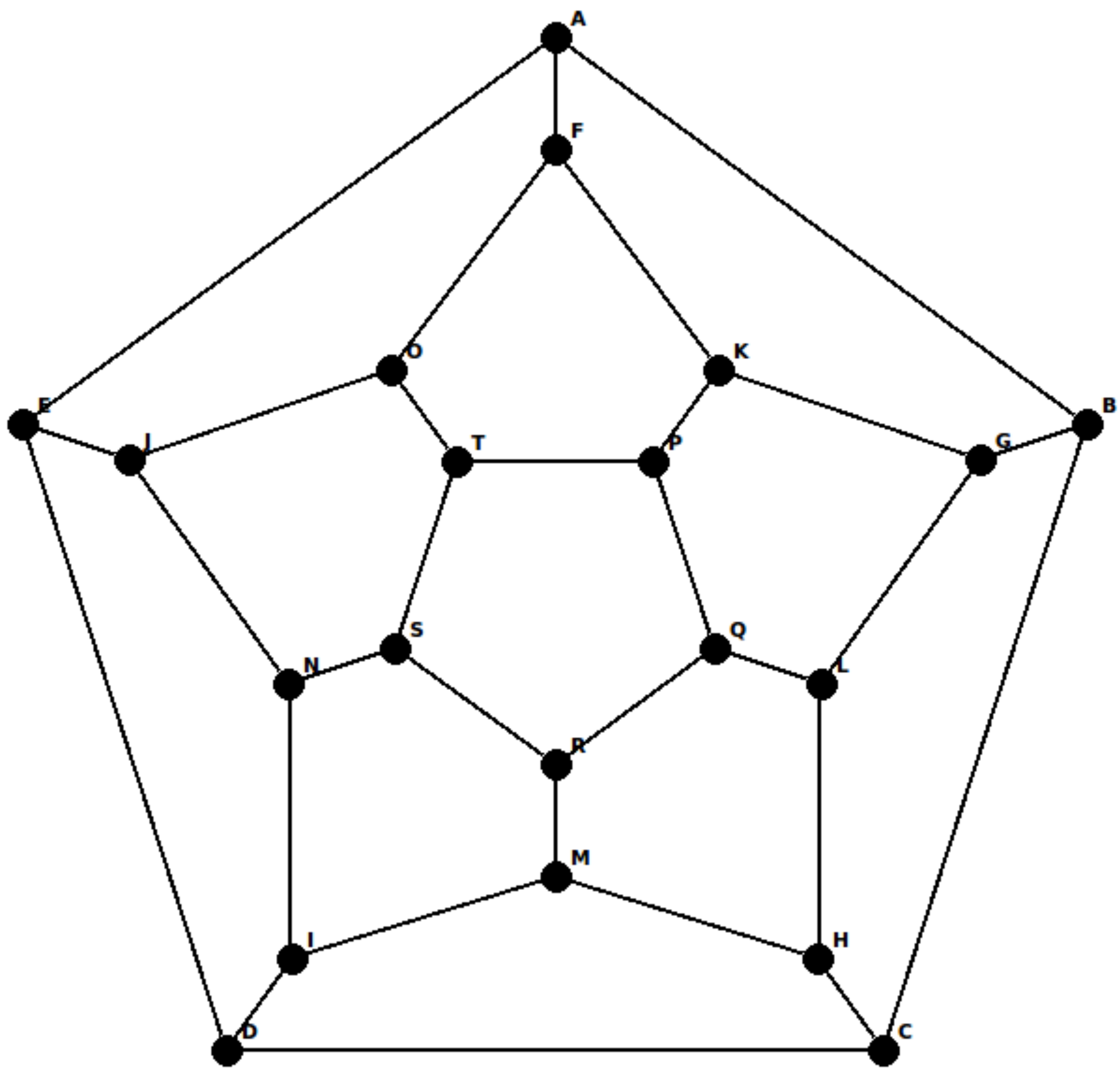
Let $G = (V, E)$ be a graph, $v \in V$, $u \in V$, and the relation $v \mathrm{~R~} u$ be connectedness. Prove that the relation $v \mathrm{~R~} u$ is an equivalence relation.

Problem 2. Fleury's Algorithm



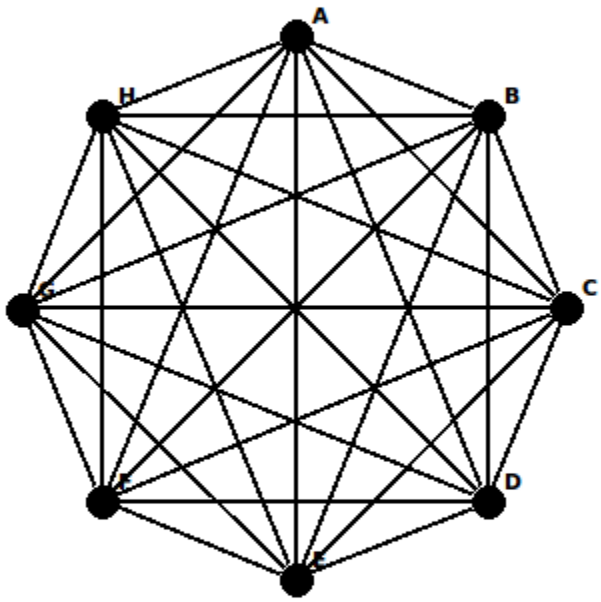
Find the Eulerian cycle for this graph using Fleury's algorithm.

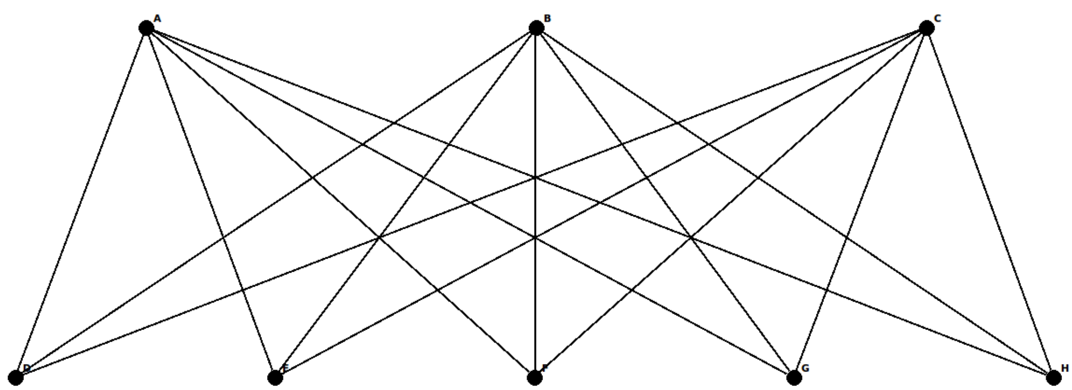
Problem 3. Hamiltonian Graphs



For this graph determine whether is has a Hamiltonian cycle or not.

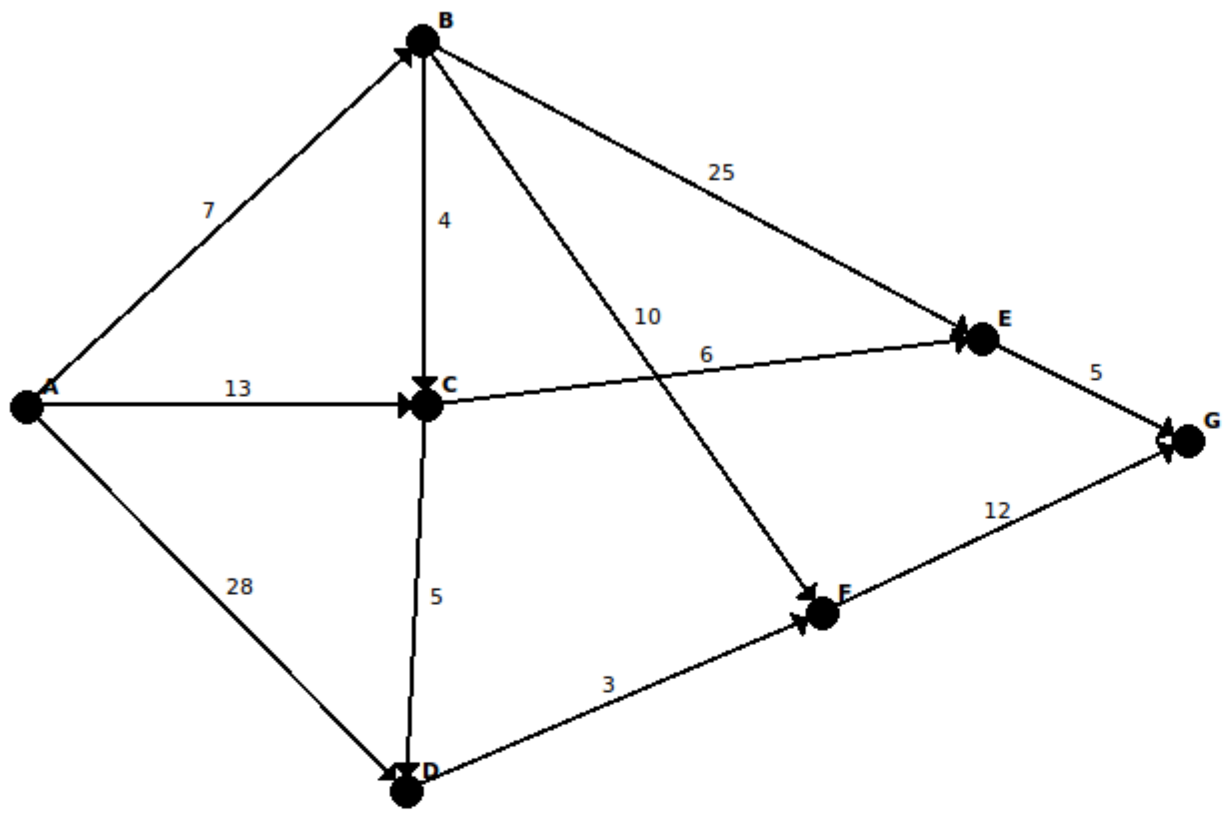
Problem 4. Types of Graphs





Name these graphs and give the corresponding symbolic representation. Reviewing the section on Bipartite Graphs on page 656 of Rosen will help.

Problem 5. Shortest Path and Scheduling



a) Find the shortest path from A to G using the tabulation method. You may also use Dijkstra's algorithm on page 709 of Rosen.
b) Treat the graph as an activity network. Construct a table giving the earliest and latest start times and float times of each activity.

Problem 6. Graph Properties

Give the diameter, radius, and chromatic number for each of the graphs in problems 2, 3, 4, and 5. For unweighed edges assume the edge weights are 1.

Problem 7. Breadth First Search

Perform a breadth first search for the graph in problem 2 starting at A . List the sequence of nodes visited and draw the resulting tree. When performing a BFS or DFS on a graph do not revisit the same node twice. In other words when a vertex has been visited remove it from the list of available vertices. You may of course backtrack to nodes you have already included in your BFS or DFS tree. Doing so avoids infinite loops.

Problem 8. Depth First Search

Perform a depth first search for the graph in problem 2 starting at A. List the sequence of nodes visited and draw the resulting tree.

Problem 9. Recursion on a Tree

Write a recursive algorithm that returns the greatest depth in a binary tree.

Problem 10. Balanced Binary Tree

Recall the definition of balanced tree from page 753 in Rosen, Example 10.
Prove that a balanced binary tree has depth less than $2\log_2(n) + 1$ by structural induction where n is the number of nodes in the tree.