Problem 1. Logic

True or false? Support your answer with an example or counter example:

(a) $\exists x \in \mathbb{Z} \ni \forall y \in \mathbb{Z}, x + y = 0$.

(b) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \ni x + y = 0$.

(c) Give an example of inference using *Modus ponens*.

Problem 2. Set Theory

Let A, B,and C be sets. Prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

Problem 3. Sums and Sequences

Given the sequence $\left\{\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625} \ldots\right\}$:

(a) Find the formula for the nth term .

(b) Give the formula for the sum of the first m terms.

Problem 4. Direct Proof

Prove that $a \mid b \wedge a \mid c \implies a \mid (b + c)$.

Problem 5. Proof by Contradiction

Prove using contradiction that the sum of a rational number and an irrational number is irrational.

Problem 6. Number Theory

(a) Decide whether $175 \equiv 22 (\text{mod } 17)$.

(b) Decide whether $3 \equiv 7 (\text{mod } 4)$.

(c) Decide whether $89 \equiv 7 (\text{mod } 4)$.

(d) Calculate the Euler Totient: $\Phi(15)$.

(e) Calculate the Euler Totient: $\Phi(77)$.

Problem 7. Encryption

Given $p = 3$, $q = 23$ and $e = 3$ show how RSA encryption would be used to encrypt and decrypt the numeric message 32.
The decrypted value is 32 and the encrypted value is 62. Those are the only value that you would not be able to compute by hand.

Problem 8. Recursion

Give a recursive algorithm for computing na using addition, where n is a positive integer and a is a real number.

Problem 9. Proof by Mathematical Induction

Use mathematical induction to show that $\sum_{j=0}^{n}(j+1) = \frac{(n+1)(n+2)}{2}$.

Problem 10. Combinatorics

Assume a standard 52 card deck for this problem, i.e. 13 distinct card values divided among 4 suits.

(a) How many 5 card combinations can be dealt?

(b) How many 5 card permutations are there?

(c) What is the probability of the first four cards dealt all being aces?