ver. 2.0
Each problem is worth 10 points.

1 Propositional Logic

1.1

Mark the propositions in the following list:

(a) Where are you?

(b) Look out for that car!

(c) North Carolina is in the United States.

(d) Load the packages onto the cart.

(e) Linux is an operating system.

(f) Jupiter is the closest planet to the Sun.

(g) if $x = 3$ then $x^2 = 9$.

(h) if $x = 3$ then $x^2 = 6$.

(i) Jane drives a Ford or Bob walks.

(j) $x + 5 = 0$.

1.2

Let p and q be the propositions:
$p =$ I will study hard.
$q =$ I will learn a lot.
Translate the following English sentences into symbols using p and q and logical operators including negations.

(a) I will study hard and I will learn a lot.

Solution

(b) I will study hard or I will learn a lot.

Solution

(c) I will not study hard and I will learn a lot.

Solution

(d) I will not study hard or I will learn a lot.

Solution

(e) If I will study hard then I will learn a lot.

Solution

(f) If I will not study hard then I will not learn a lot.

Solution

(g) I will not learn a lot therefore I will not study hard.

Solution

(h) I will not study hard therefore I will not learn a lot.

Solution

(i) I will not study hard or I will not learn a lot.

Solution

(j) I will study hard if and only if, I will learn a lot.

Solution

1.3

Let p and q be the propositions:
$p =$ Colonel Mustard, in the hall with a dagger.
$q =$ Miss Scarlett in the billiard room with a lead pipe.
$r =$ Professor Plum murdered Mr Boddy in the kitchen with the candlestick.
Translate the following propositions into English.

(a) $p \vee q$

Solution

(b) $q \Rightarrow r$

Solution

(c) $(p \vee q) \Rightarrow r$

Solution

(d) $(p \wedge q) \Leftrightarrow r$

Solution

(e) $\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Solution

1.4

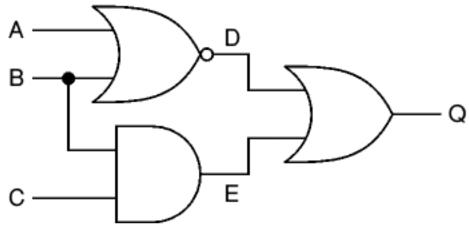
Write the truth table for $(p \wedge q) \wedge \neg (p \wedge q)$

Solution

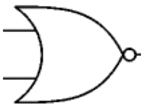
2 Applications

2.1

Give the truth table for this logic circuit and determine the propositional logic formula.



The logic gate below is a nor gate and is equivalent to $\neg(p \vee q)$



3 Equivalences

3.1

Use truth tables to show whether the following are equivalent propositions:

(a) $(p \vee q) \wedge \neg(p \wedge q)$ and $p \oplus q$

(b) $p \wedge (q \wedge r)$ and $(q \vee q) \wedge (q \vee r)$

4 Predicate Calculus

4.1

Assign symbols to represent the predicates and write the following in symbolic form:

(a) Some computers are smarter than people.

Solution

(b) For every n, there exist three integers such that the nth power of one is the sum of the nth power of the others.

Solution

(c) He is the best mathematician in the world.

Solution

(d) There are no perfect programs.

Solution

(e) For every action there is an equal and opposite reaction.

Solution

4.2

Write formal negations of the following statements:

(a) $\forall$ primes p, p is odd .

Solution

(b) $\exists$ triangle $t,$ sum of the interior angles of t is not equal to $180°$.

Solution

(c) $\forall$ politicians p, p is dishonest .

Solution

(d) $\exists$ program $p, \forall$ programs q, p can tell whether q will terminate..

Solution

(e) All people who are good at football are tall.

Solution

4.3

Find counterexamples to the following universal statements:

(a) $\forall x \in \mathbb{R}, x > \frac{1}{x}$.

Solution

(b) $\forall a \in \mathbb{Z}, \frac{(a-1)}{a} \notin \mathbb{Z}$.

Solution

(c) $\forall m, n \in \mathbb{Z}^+, mn \geq m + n$.

Solution

(d) $\forall x, y \in \mathbb{R}, \sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

Solution

(e) $\forall x, y \in \mathbb{R}, x \geq y \land z \geq x \Rightarrow z > x$.

Solution

4.4

Let $F(x, y)$ be the statement "x loves y" where the domain is everyone in the world. Use quantifiers to express each of the following:

(a) Everybody loves Tom.

Solution

(b) Everybody loves somebody.

Solution

(c) The is someone that everyone loves.

Solution

(d) There is someone who loves no one besides themselves.

Solution

(e) The is someone who loves everyone.

4.5

Write a quantified predicate that expresses the distributive law of multiplication over addition for the real numbers.

Solution

4.6

Write a quantified predicate that expresses the distributive law of conjunction over disjunction for propositional logic.

Solution

4.7

Write a quantified predicate that expresses the distributive law of conjunction over conjunction for propositional logic.

Solution

5 Logical Inference

Name the following argument forms and say whether they are valid or a fallacy. Give an example in plain English that demonstrates the argument form or fallacy.

$a \vee b$
$\neg a$
$\therefore \neg b$

Solution

$a \vee b$
$\neg a \vee c$
$\therefore b \vee c$

Solution

a
$\therefore a \vee b$

Solution

$b \wedge (a \to b)$
$\therefore a$

Solution

$\neg a \wedge (a \to b)$
$\therefore \neg b$

Solution

$\neg a \wedge (a \to b)$
$\therefore b$

Solution

$a \to b$
$\therefore \neg b \to \neg a$

Solution

$a \to b$
$\therefore \neg a \to \neg b$

Solution

$a \to b$
$\therefore b \to a$

Solution

5.1

Give English examples of argument by Modes ponens,

Solution

Modus tollens,

Solution

and Syllogism,

Solution

5.2

Use rules of inference to show that the following arguments are valid:

Solution

(a) $\neg(s \land t)$
$\neg w \to t$
$\therefore s \to w$

Solution

(b) $\neg(\neg p \lor q)$
$\neg z \to \neg s$
$(p \land \neg q) \to s$
$\neg z \lor r$
$\therefore r$

Solution

5.3

Use truth tables to show the following inference rules are valid:

(a) $p \to q$
$q \to r$
$\therefore p \to r$

Solution

(b) $p \to q$
$\neg r \to \neg q$
$\neg r$
$\therefore \neg p$

Solution

6 Introduction to Proofs

6.1

Use direct proof to show the following propositions are true:

(a) If the sum of two integers is even then so is their difference.

Solution

(b) The sum of any two rational numbers is rational.

Solution

(c) The product of any two consecutive integers is even.

Solution

6.2

Use case analysis to prove that:

(a) Consecutive integers have opposite parity (parity is whether a number is odd or even)

Solution

6.3

Use proof by contradiction to prove that there is no greatest integer.

Solution